# On a Yang-Mills type functional

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**Abstract**

In this paper we study a functional which is related with the classical Yang-Mills functional on the one way and with the Born-Infeld theory on the other way. We derive its first variation formula and prove the existence of critical points. We study conservation laws. We also obtain the second variation formula.

## Motivations

Let $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ be a smooth function. Then the graph of $u$

$$G_u = \{(x, z) \in \mathbb{R}^{n+1} \ \mid \ z = u(x), x \in \Omega\},$$

is a minimal hypersurface if and only if satisfies the following differential equation

$$\operatorname{div}\left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}\right) = 0. \tag{1}$$

In 1970 Calabi, in a paper in which he studied examples of Bernstein problems, noticed that if $n = 2$, $u$ is a $F$-harmonic map, $F(t) = \sqrt{1 + 2t} - 1$, that is $u$ is a critical point of the following functional:

$$E_F(u) = \int_{\mathbb{R}^2} F\left(\frac{\|du\|^2}{2}\right) \vartheta_g,$$

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with respect to any compactly supported variation, $\|du\|^2$ being the Hilbert-Schmidt norm.

Following the ideas of Calabi, Yang and then Sibner showed that for $n = 3$, the equation (1) is equivalent, over a simply connected domain, to the vector equation

$$\nabla \times \left( \frac{\nabla \times A}{\sqrt{1 + |\nabla \times A|^2}} \right) = 0,$$

which arises in the nonlinear electromagnetic theory of Born and Infeld. Here $A$ is a vector field in $\mathbb{R}^3$ and $\nabla \times (\ \cdot\ )$ is the curl of $(\ \cdot\ )$. Born-Infeld theory is of contemporary interest due to its relevance in string theory.

This observation leads Yang to give a generalized treatement of the equation (1), expressed in terms of differential forms, as follows

$$\delta \left( \frac{d\omega}{\sqrt{1 + \|d\omega\|^2}} \right) = 0, \tag{2}$$

for any $\omega \in A^p(\mathbb{R}^4)$. It is not very difficult to verify that the solution of equation (2) is a critical point of the following integral

$$\int_{\mathbb{R}^4} (\sqrt{1 + \|d\omega\|^2} - 1)\vartheta_g.$$

These facts gives as the motivation to study a similar functional, defined more general on Riemannian manifolds, functional which is on its definition, in some sense, similar to the well-known Yang-Mills functional.

The paper is organized as follows. In the first section we give some preliminaries and define the functional. In Section 2 we derive the Euler-Lagrange equations and give an existence result. Section 3 is devoted to a conservation law of the functional. Finally in Section 4 we derive the second variation formula.

# 1 The functional

Let $E$ be a smooth real vector bundle over a compact $n$-dimensional Riemannian manifold $(M^n, g)$, such that its structure group $G$ is a compact Lie subgroup of the orthogonal group $O(n)$.

For any vector bundle $F$ over $M$ we denote by $\Gamma(F)$ the space of smooth cross sections of $F$ and for each $p \geq 0$ we denote by $\Omega^p(F) = \Gamma(\Lambda^p T^*M \otimes F)$ the space of all smooth $p$-forms on $M$ with values in $F$. Note that $\Omega^0(F) = \Gamma(F)$.

A connection $D$ on the vector bundle $E$ is defined by specifying a covariant derivative, that is a linear map

$$D : \Omega^0(E) \to \Omega^1(E),$$

such that $D(fs) = df \otimes s + fDs$, for any section $s \in \Omega^0(E)$ and any smooth function $f \in C^\infty(M)$.

A connection $D$ is called to be a $G$-connection if the natural extension of $D$ to tensor bundles of $E$ annihilates the tensors which define the $G$-structure. We denote by $\mathcal{C}(E)$ the space of all smooth $G$-connections $D$ on $E$.

Given a connection on $E$, the map $D : \Omega^0(E) \to \Omega^1(E)$ can be extended to a generalised de Rham sequence

$$\Omega^0(E) \xrightarrow{d^D = D} \Omega^1(E) \xrightarrow{d^D} \Omega^2(E) \xrightarrow{d^D} \ldots$$

For each $G$-connection $D$ of the vector bundle $E$, the curvature tensor of $D$, denoted by $R^D$, is determined by $(d^D)^2 : \Omega^0(E) \to \Omega^2(E)$. If we suppose that $E$ carries an inner product compatible with $G$, it is easy to see that $R^D \in \Omega^2(g_E)$, where $g_E \subset \mathrm{End}(E)$ is the subbundle of skew-symmetric endomorphisms of $E$.

Given metrics on $M$ and $E$, there are naturally induced metrics on all associated bundles, such as $\Lambda^p T^*M \otimes \mathrm{End}(E)$:

$$< \varphi, \psi >_x = \sum_{1 < i_1 < \ldots < i_p < n} < \varphi^t(e_{i_1}, \ldots, e_{i_p}), \psi(e_{i_1}, \ldots, e_{i_p}) >,$$

where, for any point $x \in M$, $\{e_i\}_{i=1}^n$ is an orthonormal basis of $T_x M$ with respect to the metric $g$. The pointwise inner product gives an $L_2$-norm on $\Omega^p(E)$ by setting

$$(\varphi, \psi) = \int_M < \varphi, \psi > \vartheta_g.$$

With respect to this norm, the formal adjoint of $d^D$ it is denoted by $\delta^D$ (the coderivative) and satisfies:

$$(d^D \varphi, \psi) = (\varphi, \delta^D \psi).$$

In particular, for any $G$-connection $D$, the norm of the curvature $R^D$ is defined by

$$\|R^D\|_x^2 = \sum_{i<j} \|R^D_{e_i,e_j}\|_x^2,$$

for any point $x \in M$ and any orthonormal basis $\{e_i\}_{i=\overline{1,n}}$ on $T_xM$. The norm of $R^D_{e_i,e_j}$ is the usual one on $\mathrm{End}(E)$, namely $< A, B >= \frac{1}{2}tr(A^t \circ B)$.

Now we are able to define the Yang-Mills-Born-Infeld functional $YM_{BI} : \mathcal{C}(E) \to \mathbb{R}$ by

$$YM_{BI}(D) = \int_M (\sqrt{1+\|R^D\|^2} - 1)\vartheta_g.$$

# 2 The first variation formula. Existence result.

In the following we shall derive the Euler-Lagrange equations of the functional $YM_{BI}$.

**Theorem 1.** *The first variation formula of the functional $YM_{BI}$ is given by:*

$$\frac{d}{dt}|_{t=0} YM_{BI}(D^t) = \int_M < B, \delta^D \left( \frac{1}{\sqrt{1+\|R^D\|^2}} R^D \right) > \vartheta_g,$$

*where*

$$B = \frac{d}{dt}|_{t=0} D^t.$$

*Consequently, $D$ is a critical point of $YM_{BI}$ if and only if*

$$\delta^D \left( \frac{1}{\sqrt{1+\|R^D\|^2}} R^D \right) = 0,$$

*which are the Euler-Lagrange equations of $YM_{BI}$.*

**Proof**: Let $D$ a $G$-connection $D \in \mathcal{C}(E)$ and consider a smooth curve $D^t = D + \alpha^t$ on $\mathcal{C}(E)$, $t \in (-\epsilon, \epsilon)$, such that $\alpha^0 = 0$, where $\alpha^t \in \Omega^1(g_E)$. The corresponding curvature is given by

$$R^{D^t} = R^D + d^D\alpha^t + \frac{1}{2}[\alpha^t \wedge \alpha^t],$$

where we define the bracket of $g_E$-valued 1 forms $\varphi$ and $\psi$ by the formula $[\varphi \wedge \psi](X,Y) = [\varphi(X), \psi(Y)] - [\varphi(Y), \psi(X)]$ for any vector fields $X, Y \in \Gamma(TM)$. Indeed for any vector fields $X, Y \in \Gamma(TM)$ and $u \in \Gamma(E)$ we have:

$$\begin{aligned}
R^{D^t}(X,Y)(u) &= D^t_X(D^t_Y u) - D^t_Y(D^t_X u) - D^t_{[X,Y]}u = \\
&= D^t_X(D_Y u + \alpha^t(Y)(u)) - D^t_Y(D_X u + \alpha^t(X)(u)) \\
&\quad - D^t_X(D_{[X,Y]}u + \alpha^t([X,Y])(u)) = \\
&= D_X(D_Y u + \alpha^t(Y)(u)) + \alpha^t(X)(D_Y u + \alpha^t(Y)(u)) - \\
&\quad - D_Y(D_X u + \alpha^t(X)(u)) - \alpha^t(Y)(D_X u + \alpha^t(X)(u)) - \\
&\quad - D_{[X,Y]}u - \alpha([X,Y])(u) = \\
&= R^D(X,Y)(u) + D_X(\alpha^t(Y)(u)) - \alpha^t(Y)(D_X u) - \\
&\quad - \big(D_Y(\alpha^t(X)(u)) - \alpha^t(X)(D_Y u)\big) - \alpha^t([X,Y])(u) + \\
&\quad + \alpha^t(X)(\alpha^t(Y)(u)) - \alpha^t(Y)(\alpha^t(X)(u)) = \\
&= R^D(X,Y)(u) + (D_X(\alpha^t(Y))(u) - (D_Y(\alpha^t(X))(u) - \\
&\quad - \alpha^t([X,Y])(u) + \frac{1}{2}[\alpha^t \wedge \alpha^t](X,Y)(u) = \\
&= R^D(X,Y)(u) + (d^D\alpha^t)(X,Y)(u) + \frac{1}{2}[\alpha^t \wedge \alpha^t](X,Y)(u)
\end{aligned}$$

Then we have

$$\begin{aligned}
\frac{d}{dt}|_{t=0}(\sqrt{1 + \|R^{D^t}\|^2} - 1) &= \frac{1}{\sqrt{1+\|R^D\|^2}} \frac{d}{dt}_{|t=0} \frac{1}{2}\|R^{D^t}\|^2 = \\
&= \frac{1}{\sqrt{1+\|R^D\|^2}} < \frac{d}{dt} R^{D^t}, R^D > |_{t=0} \\
&= \frac{1}{\sqrt{1+\|R^D\|^2}} < d^D B, R^D >
\end{aligned}$$

where $B = \frac{d}{dt}|_{t=0} D^t \in \Omega^1(g_E)$.

Thus we obtain:

$$\frac{d}{dt}|_{t=0} YM_{BI}(D^t) = \int_M \frac{1}{\sqrt{1+\|R^D\|^2}} < d^D B, R^D > \vartheta_g =$$

$$= \int_M < B, \delta^D \left( \frac{1}{\sqrt{1 + \|R^D\|^2}} R^D \right) > \vartheta_g.$$

□

After we have derived the Euler-Lagrange equations, the next step is to see if the above equation has solutions. We will prove now an existence theorem for critical points of the functional $YM_{BI}$.

**Theorem 2.** *Let $(M, g)$ be an $n$-dimensional compact Riemannian manifold with $n \geq 5$, $G$ a compact Lie group, and $E$ a smooth $G$-vector bundle over $M$. Then there exists a Riemannian metric $\tilde{g}$ on $M$ conformaly equivalent to $g$ and a $G$-connection $D$ on $E$ such that $D$ is a critical point of the functional $YM_{BI}$.*

**Proof**: We prove the theorem in two steps.

**Step 1** We look at the functional $F_p : \mathcal{C}(E) \to \mathbb{R}$, defined by

$$F_p(D) = \frac{1}{2} \int_M (1 + \|R^D\|_g^2)^{(p-2)/2} \vartheta_g,$$

for which it is known that satisfies the Palais-Smale conditions and attains the minimum if $2p > n$ (see [4]). The Euler-Lagrange equation associated to this functional is

$$\delta_g^D \left( (1 + \|R^D\|_g^2)^{(p-2)/2} R^D \right) = 0.$$

This equation has a solution $D$ for $2p > n$. Define now on $M$ the function $f$ by $f = (1 + \|R^D\|_g^2)^{(p-2)/n-4}$ and the metric $\overline{g} = fg$, conformally equivalent to $g$. As $\delta_g^D \left( f^{(n-4)/2} R^D \right) = 0$ it easy to see that $\delta_{\overline{g}}^D \left( R^D \right) = 0$. Thus we have obtained that there exists a Riemannian metric $\overline{g}$ on $M$, conformaly equivalent to $g$, and a $G$-connection $D$ on $E$ such that $D$ is a Yang-Mills connection with respect to $\overline{g}$.

**Step 2** Now we look for a "good" function $\sigma$ such that $\tilde{g} = \sigma^{-1} g$. Due to the first step we can start with an Yang-Mills connection $D$ with respect to the metric $g$. It is clear that

$$\delta_g^D R^D = 0 \quad \text{if and only if} \quad \delta_{\tilde{g}}^D \left( \sigma^{\frac{n-4}{2}} R^D \right) = 0,$$

for any $G$-connection.

The function $\sigma$ is good if it satisfies the following functional equation:

$$\sigma^{\frac{n-4}{2}} = \frac{1}{\sqrt{1+\sigma^2\|R^D\|_g^2}} \left( = \frac{1}{\sqrt{1+\|R^D\|_{\tilde{g}}^2}} \right).$$

So, what we have to do now is to solve the above functional equation.

Let $h : [0,\infty) \to [0,\infty)$ given by $h(t) = \sqrt{1+2t} - 1$. It is clear that its derivative is a strictly decreasing function and let $H : (0,1] \to [0,\infty)$ its smooth inverse. We define now the smooth function $F : (0,1] \to [0,\infty)$ by

$$F(y) = \frac{H(y^{(n-4)/2})}{y^2}.$$

It is not difficult to prove that $F$ is invertible and we denote by $\Phi : [0,\infty) \to (0,1]$ the smooth inverse of $F$. We define the positive smooth function $\sigma$ by

$$\sigma = \Phi(\frac{1}{2}\|R^D\|_g^2).$$

Finally we have

$$0 = \delta_{\tilde{g}}^D \left(\sigma^{(n-4)/2} R^D\right) = \delta_{\tilde{g}}^D \left( (\Phi(\frac{1}{2}\|R^D\|_g^2))^{(n-4)/2} R^D \right) =$$

$$= \delta_{\tilde{g}}^D \left( \frac{1}{\sqrt{1+\sigma^2\|R^D\|_g^2}} R^D \right) = \delta_{\tilde{g}}^D \left( \frac{1}{\sqrt{1+\|R^D\|_{\tilde{g}}^2}} R^D \right),$$

which prove that the Yang-Mills connection $D$ is also a critical point of the functional $YM_{BI}$ with respect to the metric $\tilde{g}$. □

**Remark 1.** *The condition $n \geq 5$ is crucial in the previous proof because the Euler Lagrange equations are conformal invariant for the dimension $n = 4$.*

# 3 The stress-energy tensor. Conservation law

Motivated by the ideas of Feynman on stationary electromagnetic field, Baird and Eells introduced in 1982 the stress-energy tensor associated to any smooth map $f : (M, g) \to (N, h)$ between two Riemannian manifolds by $S_f = e(f)g - f^*h$, where $e(f)$ is the energy density of $f$. In the same spirit, it can be associated to any $G$-connection $D$ an analougue 2-tensor (related to the Yang-Mills-Born-Infeld functional) which is defined by (see also [1]):

$$S_D = (\sqrt{1 + \|R^D\|^2} - 1)g - \frac{1}{\sqrt{1 + \|R^D\|^2}} R^D \odot R^D,$$

where $R^D \odot R^D$ is the symmetric product defined by $R^D \odot R^D = < i_X R^D, i_Y R^D >$ .

It is natural to ask what is the interpretation of this tensor. There is a variational interpretation which we will try to explain in the following. Let us consider the following functional:

$$\mathcal{E}_D(g) = \int_M (\sqrt{1 + \|R^D\|^2} - 1)\vartheta_g.$$

The difference between this functional and the functional $YM_{BI}$ is that $\mathcal{E}_D$ it is defined on the space of smooth Riemannian metrics on the base manifold $M$ and the connection $D$ is fixed. We calculate now the rate of change of $\mathcal{E}_D(g)$ when the metric on the base manifold is changed. To this end, we consider a smooth family of metrics $g_s$ with $s \in (-\varepsilon, +\varepsilon)$, such that $g_0 = g$. The "tangent" vector on $g$ to the curve of metrics $g_s$ is denoted by $\delta g = \frac{dg_s}{ds}|_{s=0}$ and can be viewed as a smooth 2-covariant symmetric tensor field on $M$. Using the formulas obtained by Baird (see [1]):

$$\frac{d\|R^D\|_{g_s}}{ds}|_{s=0} = - < R^D \odot R^D, \delta g >,$$

and

$$\frac{d}{ds}\vartheta_{g_s}|_{s=0} = \frac{1}{2} < g, \delta g > \vartheta_g$$

we get :

$$\frac{d\mathcal{E}_D(g_s)}{ds}|_{s=0}$$

$$= \int_M \frac{1}{\sqrt{1 + \|R^D\|^2}} \frac{d}{ds}\left(\frac{1}{2}\|R^D\|^2\right)|_{s=0}\vartheta_g + \int_M (\sqrt{1 + \|R^D\|^2} - 1)\frac{d}{ds}\vartheta_{g_s}|_{s=0}$$

$$= \frac{1}{2}\int_M < (\sqrt{1+\|R^D\|^2}-1)g - \frac{1}{\sqrt{1+\|R^D\|^2}} R^D \odot R^D, \delta g > \vartheta_g$$

$$= \frac{1}{2}\int_M < S_D, g > \vartheta_g.$$

**Definition 1.** *A $G$-connection $D$ is said to satisfy a conservation law if $S_D$ is divergence free*

Concerning this notion we obtain the following

**Proposition 1.** *Any critical point of the functional $YM_{BI}$ is conservative*

**Proof**: The following formula for the divergence of the stress-energy tensor is true (see [3])

$$\operatorname{div} S_D(X) = \left\langle \frac{1}{\sqrt{1+\|R^D\|^2}} \delta^D R^D - i_{\operatorname{grad}\left(\frac{1}{\sqrt{1+\|R^D\|^2}}\right)} R^D, i_X R^D \right\rangle$$

$$+ \frac{1}{\sqrt{1+\|R^D\|^2}} < i_X d^D R^D, R^D >,$$

for any vector field $X$ on $M$. Using now the Bianchi identity and the Euler Lagrange equation of the functional $YM_{BI}$ we obtain that $\operatorname{div} S_D = 0$. □

# 4 The second variation formula

In this section we obtain the second variation formula of the functional $YM_{BI}$. Let $(M, g)$ be an $n$-dimensional compact Riemannian manifold, $G$ a compact Lie group and $E$ a $G$-vector bundle over $M$. Let $D$ be a critical point of the functional $YM_{BI}$ and $D^t$ be a smooth curve on $\mathcal{C}(E)$ such that $D^t = D + \alpha^t$, where $\alpha^t \in \Omega^1(g_E)$ for all $t \in (-\varepsilon, \varepsilon)$, and $\alpha^0 = 0$. The infinitesimal variation of the connection associated to $D^t$ at $t = 0$ is

$$B := \frac{d\alpha^t}{dt}|_{t=0} \in \Omega(g_E).$$

Define an endomorphism $\mathcal{R}^D$ of $\Omega^1(g_E)$ following [2] by

$$\mathcal{R}^D(\varphi)(X) := \sum_{i=1}^{n}[R^D(e_i, X), \varphi(e_i)],$$

for $\varphi \in \Omega(g_E)$ and $X \in \Gamma(TM)$, where $\{e_i\}_{i=1}^n$ is a local orthonormal frame field on $(M, g)$. Then we obtain:

**Theorem 3.** *Let $(M, g)$ be an $n$-dimensional compact Riemannian manifold, $G$ a compact Lie group and $E$ a $G$-vector bundle over $M$. Let $D$ be a critical point of $YM_{BI}$. Then the second variation of the functional $YM_{BI}$ is given by:*

$$\frac{d^2}{dt^2}|_{t=0} YM_{BI}(D^t) = -\int_M \frac{1}{(1+\|R^D\|^2)^{3/2}} < d^D B, R^D >^2 \vartheta_g +$$

$$+ \int_M \frac{1}{\sqrt{1+\|R^D\|^2}} \left(< d^D B, d^D B > + < B, \mathcal{R}^D(B) >\right) \vartheta_g =$$

$$= \int_M < B, \mathcal{S}^D(B) > \vartheta_g,$$

*where $\mathcal{S}^D$ is a differential operator acting on $\Omega(g_E)$ defined by:*

$$\mathcal{S}^D(B) = -\delta^D \left( \frac{1}{(1+\|R^D\|^2)^{3/2}} < d^D B, R^D >^2 \right) + \delta^D \left( \frac{1}{\sqrt{1+\|R^D\|^2}} d^D B \right) +$$

$$+ \frac{1}{\sqrt{1+\|R^D\|^2}} \mathcal{R}^D(B).$$

**Proof**: As $R^{D^t} = R^D + d^D\alpha^t + \frac{1}{2}[\alpha^t \wedge \alpha^t]$ and $\alpha^0 = 0$ we obtain that

$$\frac{d^2}{dt^2}|_{t=0} \left( \frac{1}{2}\|R^{D^t}\|^2 \right) = < d^D C + [B, B], R^D > + < d^D B, d^D B >,$$

where $C := \frac{d^2}{dt^2}|_{t=0} \alpha^t$. Thus we obtain:

$$\frac{d^2}{dt^2}|_{t=0} YM_{BI}(D^t) = \frac{d}{dt}|_{t=0} \int_M \frac{1}{2} \frac{1}{\sqrt{1+\|R^D\|^2}} \frac{d}{dt} \|R^{D^t}\|^2 \vartheta_g =$$

$$= -\frac{1}{4} \int_M \frac{1}{(1+\|R^D\|^2)^{3/2}} \left( \frac{d}{dt}|_{t=0} \|R^{D^t}\|^2 \right)^2 \vartheta_g + \frac{1}{2} \int_M \frac{1}{\sqrt{1+\|R^D\|^2}} \frac{d^2}{dt^2}|_{t=0} \|R^{D^t}\|^2 \vartheta_g =$$

$$= - \int_M \frac{1}{(1+\|R^D\|^2)^{3/2}} < d^D B, R^D >^2 \vartheta_g +$$

$$+ \int_M \frac{1}{\sqrt{1+\|R^D\|^2}} \left( < d^D C + [B,B], R^D > + < d^D B, d^D B > \right) \vartheta_g .$$

On the other hand, since $D$ is a critical point of the functional $YM_{BI}$, we have:

$$\int_M \frac{1}{\sqrt{1+\|R^D\|^2}} < d^D C, R^D > \vartheta_g = \int_M \left\langle C, \delta^D \left( \frac{1}{\sqrt{1+\|R^D\|^2}} R^D \right) \right\rangle \vartheta_g = 0.$$

Finally, one can prove that

$$< [B \wedge B], R^D > = < B, \mathcal{R}^D(B) > .$$

Indeed

$$< [B \wedge B], R^D > = \sum_{i<j} < [B \wedge B](e_i, e_j), R^D(e_i, e_j) > =$$

$$= \sum_{i<j} < [B(e_i), B(e_j)] - [B(e_j), B(e_i)], R^D(e_i, e_j) > =$$

$$= 2 \sum_{i<j} < [B(e_i), B(e_j)], R^D(e_i, e_j) > =$$

$$= \sum_{i,j=1}^{n} < B(e_i), [B(e_j), R^D(e_i, e_j)] > =$$

$$= \sum_{i=1}^{n} < B(e_i), \mathcal{R}^D(e_i) > =$$

$$=< B, \mathcal{R}^D(B) > .$$

and thus we obtain the second variation formula.

∎

The index, nullity and stability of a critical point of $YM_{BI}$ can be defined in the same way as in the case of Yang-Mills connection (see [2]) but is rather difficult to analyse them because the form of $\mathcal{S}^D$ is much more complicated compared with the case of Yang-Mills connections.

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