
# REAL and COMPLEX DIFFERENTIAL GEOMETRY

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**Web page of the Workshop:**

http://gta.math.unibuc.ro/pages/gvilcu/RCDG/index.html

# ABSTRACTS

## *Higher codimension CR structures, Levi-Kähler reduction and toric geometry*

**Vestislav Apostolov (Montreal)**

CR structures in codimension one play an increasingly important role in differential geometry, deeply intertwined with Kähler and Sasaki geometry. In this talk, based on a joint work in progress with D. Calderbank, P. Gauduchon and E. Legendre, I will discuss the relation between CR structures in higher codimension and Kähler geometry, through a process called "Levi– Kähler reduction". I focus in particular on the toric case, where Levi–Kähler reduction provides a new way to construct distinguished metrics on toric varieties. When the Delzant polytope is a product of simplices, explicit quotients of products of spheres are obtained, generalizing Bryant's Bochner-flat metrics on weighted projective spaces.

## *Classification of pseudo-Riemannian submersions with totally geodesic fibres from pseudo-hyperbolic spaces*

**Gabriel Bădiţoiu (Bucharest)**

We classify pseudo-Riemannian submersions with connected totally geodesic fibres from a real pseudo-hyperbolic space onto a pseudo-Riemannian manifold. Also, we obtain the classification of the pseudo-Riemannian submersions with (para-)complex connected totally geodesic fibres from a (para-)complex pseudo-hyperbolic space onto a pseudo-Riemannian manifold.

## *Invariants of conformal submanifolds*

**Florin Belgun (Bucharest)**

A submanifold $N$ in a conformal-Riemannian manifold $M$ can be considered to be "totally geodesic" if: 1. It is totally geodesic for some torsion-free conformal connection on $M$; 2. Every conformal geodesic of $M$, *strongly* tangent to $N$, is contained in $N$; or 3. Every conformal geodesic in $N$ is also a conformal geodesic in $M$. These 3 notions coincide for $M$ conformally flat, but turn out to be distinct for curved ambient spaces $M$, and correspond to the vanishing of some corresponding tensorial invariants of the embedding of $N$ in $M$. Besides these extrinsic curvature tensors, the embedding of $N$ in $M$ induces two intrinsic structures on $N$: a Laplace structure (which is a projective structure if $N$ is a curve), and a Moebius structure if dim $N>1$.

## *Extremal Sasakian Geometry: the Join and Admissible Constructions*

**Charles Boyer (Albuquerque)**

I begin the talk by giving a brief general discussion about Sasakian geometry and extremal structures in particular. I want to focus on several foundational problems: 1. Given a manifold determine how many contact structures of Sasaki type there are. 2. Given an isotopy class of contact structures determine the space of compatible extremal Sasakian structures. 3. Given extremal Sasakian structures when do they have constant scalar curvature? We concentrate our discussion on examples obtained by combining the join construction for Sasakian structures with the admissible construction for Kählerian structures. If time permits I also briefly discuss the relation with the CR Yamabe problem. My talk is based on joint work with Christina Tønnesen-Friedman.

## *On the Spectral Geometry of closed Riemannian manifolds in the presence of a smooth function*

**Dan Burghelea (Ohio)**

A smooth function on a closed Riemannian manifold $(M,g)$ provides a holomorphic family of elliptic operators $\Delta_q(z)$, $z \in \mathbb{C}$, where $\Delta_q(z)$ is a zero order perturbations of the Laplace-Beltrami operator $\Delta_q=\Delta_q(0)$. The operators $\Delta_q(z)$'s are referred to as *Witten Laplacians*. This talk reports results on the spectral theory of the families $\Delta_q(z)$.

When the smooth function $f$ is Morse then the spectral properties of $\Delta_q(z)$ for $z$ real and large enough are related to the topology of the pair $(M, grad_g f)$ (via the Witten-Helffer-Sjöstrand theory). In join work with Yoonweon Lee we explore the implications of the results about $t$ real and large on the spectral geometry of $(M,g, \Delta_q(0))$.

An interesting implication is the construction of finite dimensional sub complex of the de-Rham complex, the *virtual small sub complex* $(\Omega^*(M)_{vsm}, d^*) \subset (\Omega^*(M), d^*)$, equipped with a canonical base, invariant to $\Delta_q$. The sub complex is associated to the Morse function $f$.

The base consists of eigenforms of $\Delta_q$. These eigenforms together with the corresponding eigenvalues provide, what we call, the *virtual small spectral package*, a finite part of the infinite spectral package = all eigenvalues and eigenforms of $\Delta_q$.

One expects that interesting topological and differential geometric invariants derived from the infinite spectral package can be still recovered from the virtually small spectral package. One expects that the virtual small spectral package depends on the Morse function up to a perturbation by Morse functions.

## *Integral Pinching for the curvature*

**Gilles Carron (Nantes)**

It is a joint work with Vincent Bour (Univ. Nantes). We show some optimal Bochner's type result based on integral of curvature. Our result yields some topological restriction in dimension 3.

## *On cotangent manifolds, complex structures and generalized geometry*

**Liana David (Bucharest)**

We develop various properties of symmetric generalized complex structures (in connection with their holomorphic space and $B$-field transformations), which are analogous to the well-known results of Gualtieri on skew-symmetric generalized complex structures. Given a symmetric or skew-symmetric generalized complex structure $\mathcal{J}$ and a connection $D$ on a manifold $M$, we construct an almost complex structure $J^{\mathcal{J},D}$ on the cotangent manifold $T^*M$ and we study its integrability. Applications to Lie groups and special geometry are developed.

## *From lcK to bihermitian structures on compact complex surfaces of class* $\mathbf{VII_0}$

**Georges Dloussky (Marseille)**

Kato surfaces admit locally conformally Kähler (lcK) structures by a result of M. Brunella. We shall study conditions on this structure which imply the existence of a bihermitian structure. This is a joint work with Vestislav Apostolov and Michael Bailey.

## *Quasihomogeneous real and complex geometric structures*

**Sorin Dumitrescu (Nice)**

This talk deals with rigid geometric structures which are quasihomogeneous, in the sense that they are locally homogeneous on an open dense subset of a manifold, but not on all of the manifold. Our motivation comes from Gromov's open-dense orbit theorem and its application to prove the differential rigidity of some smooth Anosov systems. More precisely, we will present the classification of quasihomogeneous real analytic connections on surfaces (collaboration with A. Guillot) and the case of real analytic Lorentz metrics on threefolds (collaboration with K. Melnick). We will also present the corresponding classification results on complex manifolds.

## *X-ray transform on manifolds with Anosov geodesic flow*

**Colin Guillarmou (Paris)**

X-ray transform of a function is the collection of its integrals on closed geodesics. We study properties of this map for Anosov flows and the relations with the space of distributions which are invariant by the flow.

## *Compact homogeneous locally conformally Kähler manifolds*

**Paul Gauduchon (Palaiseau)**

We prove that any compact homogeneous locally conformally Kähler manifold has parallel Lee form. Based on joint work with A. Moroianu and L. Ornea.

## *Harmonic maps vs Yang-Mills fields*

**Cătălin Gherghe (Bucharest)**

We discuss several analogies and differences between Yang Mills fields and harmonic maps as solutions of variational problems.

## *Compact special Lagrangian submanifolds embedded in toric hyper-Kähler manifolds*

**Kota Hattori (Tokyo)**

In this talk, I explain the construction of compact special Lagrangian submanifolds embedded in toric hyper-Kähler manifolds by using the gluing method developed by D. Joyce.

## *Ricci surfaces*

**Andrei Moroianu (Versailles)**

A Ricci surface is a Riemannian 2-manifold $(M,g)$ whose Gaussian curvature $K$ satisfies $K\Delta K + g(dK, dK) + 4K^3 = 0$. Every minimal surface isometrically embedded in $\mathbb{R}^3$ is a Ricci surface of non-positive curvature. At the end of the 19th century Ricci-Curbastro has proved that conversely, every point $x$ of a Ricci surface has a neighborhood which embeds isometrically in $\mathbb{R}^3$ as a minimal surface, provided $K(x) < 0$. We prove this result in full generality by showing that Ricci surfaces can be locally isometrically embedded either minimally in $\mathbb{R}^3$ or maximally in $\mathbb{R}^{2,1}$, including near points of vanishing curvature. We then develop the theory of closed Ricci surfaces, possibly with conical singularities, and construct classes of examples in all genera $g \geq 2$. This is joint work with Sergiu Moroianu.

## *The geometry of gradient Ricci solitons*

**Ovidiu Munteanu (Connecticut)**

Ricci solitons are self similar solutions of the Ricci flow and model its singularities. Hence, the classification of three dimensional shrinking solitons was an important step in the proof of the Poincare conjecture. In higher dimension, the classification of solitons is a much more difficult problem and so the current efforts are focused on understanding how big the space of solitons can be. In this talk, I will discuss some progress in this direction, which concerns the topology of solitons at infinity, their volume growth rate and compactness theorems.

## *CMC proper-biharmonic surfaces of constant Gaussian curvature in spheres*

**Cezar Oniciuc (Iaşi)**

CMC surfaces in spheres are investigated under the extra condition of biharmonicity. We give a complete description of such immersions and show that for any $h \in (0,1)$ there exist CMC proper-biharmonic planes and cylinders in $\mathbf{S}^5$ with $|H| = h$, while a necessary and sufficient condition on $h$ is found for the existence of CMC proper-biharmonic tori in $\mathbf{S}^5$.

## *All the* Spin(9)*'s men*

**Maurizio Parton (Chieti-Pescara)**

The description of the Lie group Spin(9) as generated by 9 involutions in SO(16) shows that Spin(9) is the octonionic analogous of Sp(2)·Sp(1).

Starting from these 9 involutions, I will sketch out how Spin(9) is involved in several curious facts:

- there are no vertical vector fields for the octonionic Hopf fibration;
- there are ”many”" never-vanishing vector fields on spheres;
- a “matryoshka” of differential forms on Spin(7), Spin(8), Spin(9), Spin(10), Spin(12), Spin(16) can be built, leading to differential forms on the symmetric spaces FII, EIII, EVI, EVIII.

Finally, I will give a structure theorem for Spin(9) metrics which are locally conformally equivalent to Spin(9) -holonomy metrics.

*Joint work with Liviu Ornea, Paolo Piccinni, Victor Vuletescu.*

## *The fourth Severi variety and structures on some exceptional symmetric spaces*

**Paolo Piccinni (Rome)**

The Hermitian symmetric space E III = $E_6/\mathrm{Spin}(10)\cdot U(1)$ appears in F. Zak classification of smooth projective varieties which, in spite of their low codimensions, are unable to fill their ambient projective spaces through their secant and tangent lines. There are four of these *Severi varieties*, and they are the images through a *Veronese map* of the projective planes over the four algebras $\mathbb{C}$, $\mathbb{C}\otimes\mathbb{C}$, $\mathbb{C}\otimes\mathbb{H}$, $\mathbb{C}\otimes\mathbb{O}$. We discuss the construction of a canonical differential 8-form on E III, representing one of its cohomology generators and by duality related to some Schubert cycles giving a natural CW decomposition. This construction is in terms of a rank 10 even Clifford structure, a notion studied by A. Moroianu and U. Semmelmann, and can be viewed as extending the way of writing the Spin(9) canonical 8-form on F II = $F_4/\mathrm{Spin}(9)$. The present construction can be hopefully used for more canonical differential forms, in particular on the quaternion Kähler Wolf space E VI = $E_7/\mathrm{Spin}(12)\cdot\mathrm{Sp}(1)$.

*Joint work with Maurizio Parton (Università di Chieti-Pescara, Italy)*

## *On bi-Hermitian surfaces of non-Kähler type*

**Massimiliano Pontecorvo (Rome)**

We present an overview about recent results on compact complex surfaces with odd $b_1$ admitting a bi-Hermitian metric.

## *The moduli space of asymptotically cylindrical Calabi-Yau manifolds*

**Frederic Rochon (Montreal)**

We show that the examples of asymptotically cylindrical Calabi-Yau manifolds recently obtained by Haskins-Hein-Nordstrom admit a full polyhomogeneous expansion at infinity. Making use of the b-calculus of Melrose, we then establish at Tian-Todorov result in that context, namely, we show that the deformation theory of such complex manifolds is unobstructed. Time permitting, we will also discuss how to define the Weil-Petersson metric in that context as well as some of its properties. This is a joint work with Ronan Conlon and Rafe Mazzeo.

## *K-stability and parabolic stability*

**Yann Rollin (Nantes)**

Parabolic structures with rational weights encode certain iterated blowups of geometrically ruled surfaces. We show that the three notions of parabolic polystability, K-polystability and existence of constant scalar curvature Kähler metrics on the iterated blowup are equivalent, for certain polarizations close to the boundary of the Kähler cone.

## *The transverse divergence operator and taut Riemannian foliations*

**Vladimir Slesar (Craiova)**

We present a characterization of taut Riemannian foliations (*i.e.* foliated manifolds with a Riemannian bundle-like metric which turns all leaves into minimal submanifolds) using the transverse divergence operator. This result is compared with other previous techniques using colomological arguments (see J.A. Alvarez Lopez, Ann. Global Anal. Geom. 10 (1992) and H. Nozawa, J. Reine Angew. Math. 673 (2012). Furthermore, we show that a classical tautness result of Haefliger (J. Differ. Geom. 15 (1980)) can be obtained in our setting as a consequence. In the final part we present other related results.

## *On some variational problems with locally conformally Kahler background*

**Radu Slobodeanu (Neuchâtel)**

Given a mapping between Riemannian manifolds, the biharmonic energy measures its failure of harmonicity, while the σ2-energy quantifies its strength in stretching area elements. For both associated variational problems we identify classes of smooth solutions defined on l.c.K. manifolds and give explicit examples. For the σ2-problem we moreover establish the stability of some solutions defined on 4-dimensional l.c.K. manifolds. We show that this stability property is closely related to the existence of a foliation by area-stable surfaces of the domain.

## *Yamabe invariant of symplectic 4-manifolds of general type*

**Ioana Şuvaina (Vanderbilt)**

We compute the Yamabe invariant for a class of symplectic 4-manifolds obtained by taking the rational blow-down of Kähler surfaces. In particular, for any point on the half-Noether line we show that there exists a minimal symplectic manifold with known Yamabe invariant.

## *Complex subspaces of instanton moduli spaces on non-Käherian surfaces with $p_g$=0*

**Andrei Teleman (Marseille)**

We show that an instanton moduli space on a non-Käherian surfaces with $p_g$=0 contains a finite union of circles of reducible instantons (reductions). The complex structure induced by the Kobayashi-Hitchin correspondence on the stable part of the moduli space does not extend across such a circle of reductions. We give an explicit local model of the instanton moduli space around such a circle of reductions, and we study the intersection of an analytic subset of the stable part with the boundary of a tubular neighborhood of such a circle. Our result gives a strong obstruction to the existence of analytic subsets with prescribed topological behavior around the circles of reductions, and plays an important role in our program to prove existence of curves on class VII surfaces.

## *On Bott-Chern cohomology of complex manifolds*

**Adriano Tomassini (Parma)**

In this talk we will focus on various kind of cohomological decompositions on compact complex non-Kähler manifolds, involving the de Rham, Bott-Chern and Aeppli cohomologies. We also discuss some results on the cohomology of complex surfaces. The results have been obtained in joint papers with Angella, Dloussky and Verbitsky.

## *Kähler threefolds without subvarieties*

**Misha Verbitsky (Moscow)**

Let $M$ be a compact Kähler 3-fold without non-trivial subvarieties. We prove that $M$ is a complex torus. The proof is based on Brunella's fundamental theorem about structure of 1-dimensional holomorphic foliations and Demailly's regularization of positive currents. This is a joint work with F. Campana and J.-P. Demailly. I will try toexplain all notions to make the lecture accessible for anybody with basic knowledge of differential and algebraic geometry.