

**SCIENTIFIC REPORT ON THE IMPLEMENTATION OF THE
PROJECT PN-II-ID-PCE-2011-3-0118 DURING 2015**

During this period 4 scientific papers were elaborated, all being accepted or published in ISI-ranked journals. The content of these 4 papers, that cover completely the objectives proposed for this period, can be synthesized as follows:

1. Liviu Ornea, Misha Verbitsky : *Locally conformally Kähler metrics obtained from pseudoconvex shells*, Proc. Amer. Math. Soc., 144 (2016), 325–335, published electronically September 2015.

A locally conformally Kähler (LCK) manifold is a complex manifold M admitting a Kähler covering \tilde{M} , such that its monodromy acts on this covering by homotheties. A compact LCK manifold is called *LCK with potential* if its covering admits an automorphic Kähler potential. It is known that in this case \tilde{M} is an algebraic cone, that is, the set of all non-zero vectors in the total space of an anti-ample line bundle over a projective orbifold. We start with an algebraic cone C , and show that the set of Kähler metrics with potential which could arise from an LCK structure is in bijective correspondence with the set of pseudoconvex shells, that is, pseudoconvex hypersurfaces in C meeting each orbit of the associated $\mathbb{R}^{>0}$ -action exactly once. This is used to produce explicit LCK and Vaisman metrics on Hopf manifolds, generalizing earlier work by Gauduchon-Ornea and Kamishima-Ornea.

2. V. Slesar, M. Visinescu, G.E. Vilcu, *Toric data, Killing forms and complete integrability of geodesics in Sasaki-Einstein spaces $Y^{p,q}$* , Annals of Physics, Volume 361, October 2015, 548–562.

In this paper it is proved that the complete list of special Killing forms on the 5-dimensional Sasaki-Einstein spaces $Y^{p,q}$ can be extracted using the symplectic potential and the classical Delzant construction. The results achieved here agree with previous ones obtained by direct computation, proving the reliability of the method which stands in fact as a general algorithm for toric Sasaki-Einstein manifolds. Finally, we evaluate the Stäckel-Killing tensors constructed from Killing-Yano tensors and prove the complete integrability of geodesic motion in $Y^{p,q}$ spaces.

3. V. Slesar, M. Visinescu, G.E. Vilcu, *Hidden symmetries on toric Sasaki-Einstein spaces*, EPL (Europhysics Letters), Volume 110, Number 3, May 2015, 31001 (6 pp).

We describe the construction of Killing-Yano tensors on toric Sasaki-Einstein manifolds, using the fact that the metric cones of these spaces are Calabi-Yau manifolds. The description of the Calabi-Yau manifolds, using toric data allows us to find explicitly the complex coordinates and write down the Killing-Yano tensors. As a concrete example we present the complete set of special Killing forms on the five-dimensional homogeneous Sasaki-Einstein manifold $T^{1,1}$.

4. G.E. Vilcu, *On generic submanifolds of manifolds endowed with metric mixed 3-structures*, Communications in Contemporary Mathematics, (2015) 1550081 (21 pages), DOI: 10.1142/S0219199715500819.

We introduce the concept of generic submanifold in a manifold equipped with a metric mixed 3-structure and investigate the canonical distributions induced on such submanifold. In particular, we obtain necessary and sufficient conditions for the integrability of these distributions and discuss the geometry of leaves. Moreover, the paper contains a wide range of examples.

Talks at national and international conferences or in departmental seminars:

1. C. Gherghe: *On a Yang-Mills type functional*, University of Bari, 25-26 June 2015.
- 2 L. Ornea: *Compact pluricanonical manifolds are Vaisman*, "Feodor Bogomolov" Laboratory of Algebraic Geometry, Higher School of Economy, Moscow, April 2015.
3. L. Ornea: *CR manifolds in LCK geometry*, Dépt. de Mathématiques, Université libre de Bruxelles, October 2015.
4. V. Vuletescu: *Non-Kähler manifolds: Locally conformally Kähler metrics and vector bundles*, Institut de Physique Théorique, Paris, 02.06.2015.
5. V. Vuletescu: *A gentle introduction to LCK geometry*, Scuola Normale Superiore, Pisa, 10.07.2015.
6. V. Vuletescu: *Recent results in locally conformally (Kähler) geometry*, Univ. Sapienza, Dipartimento "Guido Castelnuovo", Roma, 16.07.2015.

Project manager,
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