SCIENTIFIC REPORT ON THE IMPLEMENTATION OF THE PROJECT PN-II-ID-PCE-2011-3-0118 DURING 2016

During this period 7 scientific papers were elaborated, 3 of them being accepted or published in ISI-ranked journals. The content of these 7 papers, that cover completely the objectives proposed for this period, can be synthesized as follows:

1. L. Ornea, V. Slesar: *Basic Morse-Novikov cohomology for foliations*, Mathematische Zeitschrift 284 (2016), no. 1-2, 469-489.

In this paper we find sufficient conditions for the vanishing of the Morse-Novikov cohomology on Riemannian foliations. We work out a Bochner technique for twisted cohomological complexes, obtaining corresponding vanishing results. Also, we generalize for our setting vanishing results from the case of closed Riemannian manifolds. Several examples are presented, along with applications in the context of l.c.s. and l.c.K. foliations.

2. L. Ornea, M. Verbitsky: *LCK rank of locally conformally Kaehler manifolds with potential*, J. Geom. Physics. 107 (2016), 92-98.

An LCK manifold with potential is a quotient of a Kähler manifold X equipped with a positive Kähler potential f, such that the monodromy group acts on X by holomorphic homotheties and multiplies f by a character. The LCK rank is the rank of the image of this character, considered as a function from the monodromy group to real numbers. We prove that an LCK manifold with potential can have any rank between 1 and $b_1(M)$. Moreover, LCK manifolds with proper potential (ones with rank 1) are dense. Two errata to our previous work are given in the last section.

3. M. Parton, P. Piccinni, V. Vuletescu: *Clifford systems in octonionic geometry*, Rend. Sem. Mat. Torino, to appear.

We give an inductive construction for irreducible Clifford systems on Euclidean vector spaces. We then discuss how this notion can be adapted to Riemannian manifolds, and outline some developments in octonionic geometry.

4. Jae Won Lee, Chul Woo Lee, G.E. Vîlcu: *Optimal inequalities for the normalized δ-Casorati curvatures of submanifolds in Kenmotsu space forms*, Advances in Geometry, to appear.

We establish two sharp inequalities for the normalized δ-Casorati curvatures of submanifolds in a Kenmotsu space form, tangent to the structure vector field of the ambient space. Moreover, we show that in both cases, the equality at all points characterizes the totally geodesic submanifolds and some non-trivial examples are given.

5. G.E. Vîlcu: *Paraquaternionic CR-submanifolds, mixed 3-structures and semi-Riemannian submersions*, Chapter 13 in: *Geometry of Cauchy-Riemann Submanifolds*, Editors: Sorin Dragomir, Mohammad Hasan Shahid, Falleh R. Al-Solamy, Springer (2016), 361–390.

In this work we review basic results concerning several types of submanifolds and semi-Riemannian submersions involving manifolds endowed with paraquaternionic and mixed 3-structures.

6. L. Ornea, M. Verbitsky: *Hopf surfaces in locally conformally Kaehler manifolds with potential*, preprint 2016.

We prove that every compact locally conformaly Kaehler manifold with potential contains an holomorphically embedded Hopf surface.

7. L. Ornea, M. Verbitsky, V. Vuletescu: *Weighted Bott-Chern and Dolbeault cohomology for LCK-manifolds with potential*, preprint 2016.

A locally conformally Kaehler (LCK) manifold can be seem as of a complex manifold M with a Kaehler form taking values in a local system L, called the conformal weight bundle. The L-valued cohomology of M is called Morse-Novikov cohomology. It was conjectured that (just as it happens for Kaehler manifolds) the Morse-Novikov complex satisfies the dd^c - lemma. We prove that dd^c - lemma is true with coefficients in a sufficiently general power L_a of L on any LCK manifold with potential (this includes Vaisman manifolds). We also prove vanishing of Dolbeault and Bott-Chern cohomology with coefficients in L_a and degeneration of the Dolbeault-Frohlicher spectral sequence with coefficients in any power of L.

Talks at international conferences or in departmental seminars:

1. L. Ornea: *New results in LCS geometry*, Bruxelles, Belgium (ULB), 21-29.01.2016.

2. L. Ornea: *LCK geometry*, Pohang, South Korea, Centre for Geometry and Physics, POSTECH, 14-28.02.2016.

3. L. Ornea: *The rank of locally conformally Kaehler manifolds*, plenary talk at the workshop *Special Hermitian metrics on non-Kaehler manifolds*, Firenze, Italy, 19-23.04.2016.

4. L. Ornea: *Locally conformaly Kaehler manifolds with potential*, plenary talk at the workshop *Complex and symplectic geometry*, Cortona, Italy, 12-18.06.2016.

5. L. Ornea: *The embedding problem in differential geometry*, University of New Mexico, Albuquerque, 05.09.2016.

6. L. Ornea: *Locally conformally Kaehler manifolds with potential*, Courant Institute, NY, 10.09.2016.

7. L. Ornea: *Localy conformally Kaehler manifolds with potential*, Univ. Hanovra, 28.11.2016.

8. G.E. Vîlcu: *Canonical foliations and submersions in paraquaternionic-like geometries*, Asian Mathematical Conference (AMC) 2016, Bali Nusa Dua Convention Center, 25-29 July 2016, Bali, Indonezia.

9. G.E. Vîlcu: *Special classes of submanifolds in quaternionic-like geometries*, International Conference on Differential Geometry, Algebra and Analysis (ICDGAA-16), November 15-17, 2016, New Delhi, India.

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