SCIENTIFIC REPORT ON THE IMPLEMENTATION OF THE PROJECT PN-II-ID-PCE-2011-3-0118 IN 2013

During this period seven scientific papers were elaborated, six among them being accepted or published in ISI-ranked journals and one paper is under review. Moreover, the article [C. Gherghe, *Harmonic maps and stability on locally conformal Kähler manifolds*], submitted for consideration for a possible publication in 2012, was accepted and published in [*Journal of Geometry and Physics*, 70 (2013), 48–53]. The content of these seven papers realized in 2013, that cover completely the objectives proposed for the second phase, can be synthesized as follows:

1. L. Ornea, M. Parton, V. Vuletescu: *Holomorphic submersions of locally conformally Kähler manifolds*, Annali di Matematica Pura ed Applicata, DOI 10.1007/s10231-013-0332-z, in press.

This paper was realized as part of Objective 5 in the work plan. We mention that in this paper we obtained a more general result than the initial objective. Precisely, even if in the beginning we aimed at studying only elliptic fibrations, in the paper we obtained a non-existence result for LCK metrics on total spaces of holomorphic submersions - and hence not only for locally trivial bundles, and not only for one-dimensional fibres.

The main result is the following. For a complex-analytic submersion between compact manifolds (with strictly positive dimensional fibres) such that one fibre is of Kähler type, then the domain of the submersion cannot bear LCK metrics but in the case of elliptic fibres.

Important corollaries are:

- a new proof of a result from 1999 by K. Tsukada (*The canonical foliation of a compact generalized Hopf manifold*, Differential Geom. Appl. 11 (1999), no. 1, 13–28), who showed that the product of two compact regular Vaisman maniffolds admits no LCK metric;

- a proof for the fact that the product of a LCK (not GCK) manifold with a Kähler one admits no LCK metric.

In connection with this second corollary, we note that the problem of existence of LCK metrics on the product of two LCK manifolds is still open. The above result suggests a negative answer. We aim to further investigate this problem.

2. G. E. Vîlcu: *Mixed paraquaternionic 3-submersions*, Indagationes Mathematicae, 24(2) (2013), 474–488.

In this paper a new class of semi-Riemannian submersions is defined from a manifold endowed with a metric mixed 3-structure onto an almost paraquaternionic hermitian manifold. We obtain some fundamental properties, discuss the transference of structures and the geometry of the fibres. In particular we obtain that such a submersion is a harmonic map, provided that the total space is mixed 3-cosymplectic or mixed 3-Sasakian. In the last part of the paper, some non-trivial examples are given.

3. G. E. Vîlcu: *On Chen invariants and inequalities in quaternionic geometry*, Journal of Inequalities and Applications 2013, 2013:66.

The theory of Chen invariants, was initiated by Prof. B.-Y. Chen in a paper published in 1993 [*Some pinching and classification theorems for minimal submanifolds*, Arch. Math. 60, 568-578]. The author's original motivation to introduce new types of Riemannian invariants, known as δ-invariants or Chen invariants, was the need to provide answers to an open question raised by S.S. Chern concerning the existence of minimal immersions into a Euclidean space of arbitrary dimension. In fact, due the lack of control of the extrinsic properties of the submanifolds by the known intrinsic invariants, no solutions to Chern's problem were known before the invention of Chen invariants. Therefore, in the above quoted paper, B.-Y Chen obtained a necessary condition for the existence of minimal isometric immersion from a given Riemannian manifold into Euclidean space and established a sharp inequality for a submanifold in a real space form using the scalar curvature and the sectional curvature (both being intrinsic invariants) and squared mean curvature (the main extrinsic invariant).

In this paper, a lot of extensions of Chen inequalities in quaternionic setting are given and the equality cases in the above inequalities are investigated and completely characterized. In the last part of the article, a set of natural problems in the field is proposed.

4. G. E. Vîlcu: *Canonical foliations on paraquaternionic Cauchy-Riemann submanifolds*, Journal of Mathematical Analysis and Applications, 399(2) (2013), 551–558.

Recently, in [S. Ianuş, S. Marchiafava, G.E. Vîlcu, Cent. Eur. J. Math. 8 (2010), no. 4, 735-753], the authors introduced paraquaternionic CR-submanifolds of a paraquaternionic Kähler manifold, stating some basic results on their differential geometry. On the other hand, it is known that the natural product of two paraquaternionic Kähler manifolds does not become a paraquaternionic Kähler manifold, but it is an almost paraquaternionic Kähler product manifold. It is the main aim of this paper to extend the concept of paraquaternionic CR-submanifolds in a more general setting, namely for a paraquaternionic Kähler product manifold. Firstly, the integrability of the distributions involved in the definition of the CR-submanifold is investigated. Then, the canonical foliations induced on paraquaternionic CR-submanifolds are studied. Moreover, necessary and sufficient conditions for paraquaternionic CR-submanifolds of almost paraquaternionic Kähler product manifolds to be ruled submanifolds with respect to the canonical foliations are obtained. In the last part of the paper characterizations are provided for these foliations to become semi-Riemannian, i.e. with bundle-like metric.

5. L. Ornea, M. Parton, P. Piccinni, V. Vuletescu: *Spin(9) geometry of the octonionic Hopf fibration*, Transformation Groups, 18, No. 3, 845–864 (2013)

This paper belongs to the objective of extending notions and results from LCK geometry to the more general framework of geometries locally conformal with interesting geometric structures, such as symplectic or with special holonomies (Objective 4 in the work plan). In the mentioned paper we treat the case of metrics locally conformal with metrics with special holonomy Spin(9).

The main result therein is that locally conformal Spin(9) manifolds are quotients of the 15-dimensional sphere. In other words, these manifolds are analogues of regular Vaisman manifolds from LCK geometry. Moreover, we present non-trivial examples of such manifolds, rather difficult to obtain as the study of Spin(9) is intimately connected to the octonionic algebra (which is non-associative and hence much more difficult to study than the complex numbers one appearing in LCK geometry).

Moreover, in this paper we obtained a new proof of a very deep 1992 result: B. Loo and A. Verjovsky, *The Hopf fibration over S8 admits no S1-subfibration*, Topology, 31(2):239–254.

6. A.M. Ionescu, V. Slesar, M. Visinescu, G.E. Vîlcu: *Transversal Killing and twistor spinors associated to the basic Dirac operators*, Reviews in Mathematical Physics, vol. 25, Issue 08, September 2013, 21 pp.

The main goal of this paper was to obtain the corresponding interplay between transversal Killing spinors and basic Dirac operator in the framework of Riemannian foliations. In order to made such an achievement in the general setting of Riemannian foliations we define transversal Killing spinors as natural extension of basic spinors transversally parallel with respect to a modified connection associated with the basic Dirac operator. We also introduce in a natural way a class of twistor spinors. For general Riemannian foliations these definitions differs from the classical ones, but for the particular case of Riemannian foliation with basic-harmonic curvature, which is the most convenient setting, this definition coincide with the previous definition of N. Ginoux and G. Habib, and our results turn out to be generalization of [S. D. Jung, *J. Geom. Phys.* 39 (2001) 253–264.] Moreover, the absolute case (when the manifold is foliated by points) becomes a generalization of the case of closed Riemannian manifolds. In the final part of the paper we point out some possible applications of the results and some physical considerations.

7. Cătălin Gherghe: *Harmonicity and spectral theory on Sasakian manifolds* (preprint, 2013).

The theory of harmonic maps between Riemannian manifolds endoweed with some special structures has its origin in a paper of Lichnerowicz [*Applications harmoniques et varietes Kähleriennes*, Symposia Mathematica 3(1980), 341-402] in which he considered holomorphic maps between compact Kähler manifolds. He proved that such a map is not only a harmonic map but also a minimizer of the energy functional in its homotopy class.

Ianuş and Pastore developed a theory of harmonic maps between manifolds endowed with almost contact metric structures [*Harmonic maps on contact metric manifolds*, Annales Math. Blaise Pascal, 2(1995), 43-53]. Following the ideas of Rawnsley, they introduced a notion analogous to holomorphy. Using similar tools Chinea studied submersions between almost contact metric manifolds [*Harmonicity on maps between almost contact metric manifolds*, Acta Math. Hungar. 126 (2010), no. 4, 352-362]. In Section 3 we prove an analougue result to that obtained by Lichnerowicz but in the case when the target manifold is Sasakian.

The Laplace-Beltrami operator of a compact Riemannian M can be viewed as the Jacobi operator of a constant harmonic map from M into a unit circle This is a good reason to study the spectral geometry of the Jacobi operator of a harmonic map. This was done for Kähler manifolds with constant holomorphic curvature by Urakawa [*Spectral Geometry of the second variation operator of harmonic maps*, Illinois J.Math., 33(1989), 250-267] and for Sasakian manifolds with constant φ-sectional curvature by Kang and Kim [*On the Spectral Geometry for the Jacobi Operators of Harmonic maps into a Sasakian or Cosymplectic Space form*, Comm. Korean Math. Soc. 12 (1997), No. 2, pp. 373-382.]. We will prove in Section 4 that the spectrum of the Jacobi operator associated to a harmonic map determines the geometric properties as those of harmonic morphisms when the target manifold is a sasakian space-form.

Harmonic morphism are maps which pull back germs of real valued harmonic functions on the target manifold to germs of harmonic functions on the domain. In Section 5 We prove a characterization theorem for harmonic morphisms defined on Sasakian manifolds to a Hermitian manifold. A similar type of result was obtained by Gudmundsson and Wood but in the complex case [*Harmonic morphisms between almost Hermitian manifolds*, Boll. U.M.I. 11-B (1997) suppl. fas. 2, 185-197].

It is noteworthy that the dissemination of results was performed not only by publishing articles, but also by talks at national and international conferences or in departmental seminars.