SCIENTIFIC REPORT OF THE GRANT
PN–II–ID–PCE–2011–3–1023, NR. 247/2011,
IN THE PERIOD 01.01.2012 – 30.11.2013

INTRODUCERE

We will sketch in this report the scientific activity held in the period 01.01.2012 – 30.11.2013 by the members of the grant PN–II–ID–PCE–2011–3–1023, nr. 247/2011, whose director is Prof. Dr. Dorin Popescu. The structure of the report is as follows: in Section 1 we are going to detail the scientific results obtained in this period, while in Section 2 we are going present the way that these results achieved under the finance of this grant were disseminated at the conferences/stages of research abroad. We also want to point out that a summary of the activities held within this grant is available at the following website of this grant

`https://dl.dropboxusercontent.com/u/112281424/grantPCE-2011-3-1023/indexDP.html`

Regarding the scientific results we would like to mention that up to this moment the research team of this grant wrote altogether 18 papers, 9 each year. More precisely, in the year 2012 were written the papers [2, 18, 21, 20, 43, 44, 46, 9, 10], from which so far 7 were published and the other 2 accepted for publication in prestigious ISI journals. In the year 2013 there were written the papers [8, 30, 45, 47, 48, 23, 17, 22, 35], out of which 3 were accepted for publication (see [47, 23, 22]) in ISI journals, and the others are submitted for publication to ISI journals. So overall, out of the 18 research papers written with funding from this grant 12 have been already published or accepted for publication in ISI journals (see the bibliography for details of the journals). Section 1 represents a summary of the main results obtained in these 18 research papers. It is important to mention that all the papers written so far are in the areas of research from the research proposal of this grant. These papers can be split into 2 main categories: 10 of them regard particular positive cases of Stanley's conjecture, while the other 8 are in the field of combinatorics in commutative algebra, some of them also with applications in algebraic statistics. In what concerns the dissemination of the scientific results obtained within this grant, the team members had several talks in the scientific seminar organized weekly by IMAR and Faculty of Mathematics and Computer Science (University of Bucharest), see for historic of the years 2012, 2013

`http://www.imar.ro/organization/activities/archive/seminars_arh_sem_19_s.php`, and also gave several talks: 1) at national/international conferences, and 2) scientific seminars of the universities where they were invited researchers (see Section 2 for details).

1. DESCRIPTION OF THE SCIENTIFIC RESULTS

Next we briefly describe the scientific results obtained in the 18 research papers. For a clear evidence of the year when the papers were written, we firstly present the papers finalized in the year 2012, and secondly those finalized in the year 2013. As a standard common notation for the description of all results we denote by S the polynomial ring $K[x_1,\ldots,x_n]$ in n indeterminates over the field K.

2012: In the paper [2] from references, one characterizes the unmixed monomial ideals $I \subset S = K[x_1,\ldots,x_n]$ with $\operatorname{depth}(S/I) = \operatorname{depth}(S/\operatorname{rad}(I))$, that is, those ideals of maximal depth. The depth of S/I is maximal since it is known that the Betti numbers decrese by passing to the radical of I, therefore the depth of S/I increases when we pass to $\sqrt{I}$. The characterization obtained in [2] extends results of [40]. In addition, as an application of the main result, a class of simplicial complexes called "with rigid depth" is characterized. We say that a pure simplicial complex has *rigid depth* if for every unmixed monomial ideal $I \subset S$ with $\sqrt{I} = I_\Delta$ one has $\operatorname{depth}(S/I) = \operatorname{depth}(S/I_\Delta)$. The rigid depth simplicial complexes generalize in a natural way the simplicial complexes studied by J. Herzog, Y. Takayama, N. Terai in [29]. In particular, from this characterization, it follows that if a pure simplicial complex has rigid depth over a field of characteristic 0, then it has rigid depth over any field. In the last part, the behavior of rigid depth in connection to the skeletons of the simplicial complex is discussed.

In [18], one considers ideals generated by general sets of m-minors of an $m \times n$-matrix of indeterminates. The generators are identified with the facets of an $(m-1)$-dimensional pure simplicial complex. The ideal generated by the minors corresponding to the facets of such a complex is called a determinantal facet ideal. When $m = 2$, which means that Δ is a graph, the ideal J_Δ is a binomial edge ideal. These binomial ideals were introduced in [28] and were intensively studied in the recent years. The determinantal facet ideals have a much more complicated structure than the binomial edge ideals. In [18], we discuss the question when the generating minors of its determinantal facet ideal J_Δ form a Gröbner basis and when J_Δ is a prime ideal. It is shown that the generators form a Gröbner basis with respect to the lexicographic order induced by the natural order of the variables if and only if Δ is closed, a combinatorial property which generalizes somehow the closed graphs. When Δ is closed, it is shown that J_Δ is Cohen-Macaulay and the K–algebra generated by the generators of J_Δ is Gorenstein. For Δ closed, a necessary condition for the primality of J_Δ is given. This condition is expressed in terms of combinatorics of Δ. Under additional conditions on Δ sufficient conditions for the primality of J_Δ are given.

In the paper [21], we introduce a new class of ideals of 2-minors associated with graphs. Let $X = (x_{ij})$ be an $n \times n$-matrix of variables and $S = K[X]$ the polynomial ring over a field

K in the variables $\{x_{ij}\}_{1\leq i,j\leq n}$. Let G be a simple graph on the vertex set $[n]$. With this graph we associate an ideal generated by diagonal 2-minors of X in the following way. For $1\leq i<j\leq n$ we denote by f_{ij} the diagonal 2-minor of X given by the elements at the intersections of the rows i,j and the columns i,j, that is, $f_{ij}=x_{ii}x_{jj}-x_{ij}x_{ji}$. Let P_G be the ideal of S generated by the binomials f_{ij} where $\{i,j\}$ is an edge of G.

It is shown that P_G is a prime ideal, and the ring $R_G=S/P_G$ is a normal domain. In the last section we study the divisor class group $\mathrm{Cl}(R_G)$. We show that $\mathrm{Cl}(R_G)$ is free and we express its rank in terms of the graph's data. Finally, we give sharp bounds for the possible rank of $\mathrm{Cl}(R_G)$ when G has a given number of edges. It is known that every abelian group is the class group of a Krull domain. By using ideals generated by diagonal 2-minors, one may find an example of a normal domain with free divisor class group of any given rank.

In [20] it is defined a functor $\mathfrak{r}^*$ from the category of positive determined modules to the category of the squarefree modules which plays a similar role to taking the radical for monomial ideals. As it was explained in [29], the Betti numbers do not increase when one passes from a monomial ideal to its radical. We show that passing from a positively $\mathbf{t}$-determined module to its "radical" module has a similar behavior. In particular, one obtains $\operatorname{depth} M\leq \operatorname{depth}\mathfrak{r}^*M$ for any positively $\mathbf{t}$-determined module M.

Unlike the monomial case, for a positively $\mathbf{t}$-determined module M, we show that one has only the inequality $\dim\mathfrak{r}^*M\leq\dim M$. Easy examples show that the inequality may be strict. By using the inequalities between depth and Krull dimension, we show that the (sequentially) Cohen-Macaulay property of M passes to the "radical" of M for any positively $\mathbf{t}$-determined module M with $\mathfrak{r}^*M\neq 0$. Moreover, the connections between the functor $\mathfrak{r}^*$ and the functors Ext, the Alexander dual (first introduced and studied in [39]) and the Auslander-Reiten translate [5]. The connection between r^* and Ext allows to prove that if M and $\mathfrak{r}^*(M)$ have the same Krull dimension, then M is generalized Cohen-Macaulay if and only if $r^*(M)$ is so and if M is Buchsbaum, then the radical of M is also Buchsbaum.

The papers [43, 44, 46] present important results on Stanley's conjecture for the squarefree monomial ideals. In the paper [43], it is considered the case of a monomial ideal generated by r squarefree monomials of degree d. The author proves that if r is greater or equal than the number of squarefree monomials of I of degree $d+1$, then $\operatorname{depth} S/I=d$. If J is a nonzero monomial ideal properly contained in I, generated by squarefree monomials of degree greater or equal than $d+1$, and r is strictly bigger than the number of squarefree monomials of I/J of degree $d+1$ (or more generally $\operatorname{sdepth} I/J=d$) atunci $\operatorname{depth} I/J=d$. In particular, the author obtains in the situations described above a positive answer for Stanley's conjecture. The main result (Theorem 2.2) gives a sufficient condition, namely $\rho_d(I)>\rho_{d+1}(I)-\rho_{d+1}(J)$, that implies $\operatorname{depth} I/J=d$. Here, $\rho_d(I)$ represents the number of all squarefree monomials of degree d of I. The proof of this result makes use of Koszul homology, a new technique to tackle this conjecture, introduced by the author. Moreover, the author explains why this technique seems to be better suited for this particular cases of the conjecture. If I is generated by at least $\rho_{d+1}(I)$ squarefree monomials of degree d, then it is proved in Corollary 3.4 that $\operatorname{depth} I = d$. This generalizes a previous result of the same author, the starting point for this research paper. In addition, it is also shown that the imposed conditions are consequences of the fact that $\operatorname{sdepth} I/J = d$, which means that Stanley's conjecture holds in this case.

In the paper [44], the author generalizes Theorem 2.2 from [43] in the following way. It is considered the case when I and J are two squarefree monomial ideals such that J is properly contained in I and with the property that I is generated by monomials of degree greater or equal than d, while J is generated by monomials of degree greater than or equal to $d+1$. The main result of this paper (Theorem 1.3) states that in certain conditions one can effectively compute $\operatorname{depth} I/J$ (an invariant which is in general hard to compute) which could imply a proof of Stanley's conjecture in the case of quotients of squarefree monomial ideals. In the paper [46], the authors prove new cases when Stanley's conjecture holds true. More precisely, they consider the case when I and J are two squarefree monomial ideals such that J is properly contained in I, the minimal generators of I are of degree ≥ 1, and J in degree ≥ 2. In addition, if I contains exactly one variable among the minimal generators, and the other generators are of degree greater than or equal to 2 then $\operatorname{sdepth} I/J \leq 2$ implies that $\operatorname{depth} I/J \leq 2$, thus in particular Stanley's conjecture holds true. In order to prove the main result, Theorem 1.10, the authors extend the previous results and techniques from [43, 44].

The papers [9, 10] are centered on the analysis of very particular cases of Stanley's conjecture. In the paper [9], it is proved that if I is an almost complete intersection monomial ideal then Stanley's conjecture holds for S/I and I. This result represents a non-trivial generalization of the similar known results for monomial complete intersection ideals. In the paper [10] are given sharp bounds for the Stanley depth of the quotient I/J, for two complete intersection monomial ideals I and J. As a particular case, it is computed the Stanley depth for the quotient of two irreducible monomials ideals. In addition, the author proves several inequalities regarding Stanley depth.

2013: Tn the works [23], [17] and [22] we study classes of binomial ideals associated with combinatorial objects. Binomial edge ideals were recently introduced by J. Herzog and his collaborators in [28] and have been intensively studied in the last years. For a simple graph G on the vertex set $[n]$, one considers the associated binomial edge ideal J_G in the polynomial algebra $S = K[x_1,\ldots,x_n,y_1,\ldots,y_n]$ over a field K. This is generated by all the binomials $f_{ij} = x_iy_j - x_jy_i$ with $\{i, j\}$ edge in G. These ideals have applications in statistics as shown in [4], [28], [51]. In the last years, the homological and algebraic invariants of binomial edge ideals as well as extensions of these ideals have been studied: [14], [19], [37], [42], [50], [53], [54], [56], [57].

As for any graded ideal arising from cobinatorics, one aims at expressing the homological and algebraic invariants of J_G in terms of the combinatorail data of the graph G. So far, several proerties of binomial edge ideals are known. It was shown that they are radical ideals and the minimal prime ideals can be expressed in terms of the combinatorics of G [28]. Classes of graphs for which the associated ideals are Cohen-Macaulay or Gorenstein are known [16]. A special interest is devoted to computethe regularity and the pojective dimension. In a recent paper [37] it is shown that, in general, for a connected graph G, the reularity of S/J_G is bounded above by the number of the vertices of G minus 1 and below by the length of the longest induced path in G. Moreover, it is conjectured that the maximal regularity is obtained for the path graph. In the paper [23] it is shown that if G is a closed connected graph (that staisfies a certain combinatorial condition which, from algebraic point of view is equivalent to the fact that J_G has a quadratic Gröbner basis) the regularity of S/J_G is equal to the length of the longest induced path in G. Therefore, Matsuda-Murai conjecture is true for closed graphs. In the same paper, it is shown that this conjecture is true for aclass of graphs which includes the trees. In addition, it is shown that for G closed, the regularity of J_G coincides with the regularity of the initial ideal of J_G with respect to the lexicographic order. This result supports a recent conjecture [16] which claims that J_G and its initial ideal with respect to lex order have the same extremal Betti numbers. In this direction there are just a few results. The paper [23] is an important step in this direction.

A Kosul algebra is a standard graded algebra over a field whose maximal ideal has a linear resolution. Koszul algebras appear quite often among the toric ideals and some other algebras studied in combinatorial commutative algebra and algebraic geometry [3], [12], [52]. It is known that a Koszul algebra is defined by quadrics. In addition, it is well-known that if the defining ideal of a K–algebra has a quadratic Gröbner baiss, then the algebra is Koszul.

In the paper [17], we consider K–algebras defined by binomial edge ideals, that is, of the form $R = S/J_G$. Since $J_G$is generated by quadrics, it nturally arrises the following question: For which graphs G is the algebra S/J_G Koszul? When S/J_G is Koszul, we say that the graph G is Koszul (over the field K). If G is closed, that is, J_G has a quadratic Gröbner basis, it follows that S/J_G is Koszul. Therefore, any closed graph is Koszul. On the other hand, one may easily find examples of Koszul graphs which are not closed. In [17] we show that any Koszul graph is chordal and claw-free. Consequently, we have the following implications

$$G = \text{closed graph} \Rightarrow G = \text{Koszul} \Rightarrow G = \text{chordal and claw-free}.$$

We show that none of the above implications may be reversed and we classify all the Koszul graphs for which the clique complex has dimension at most 2.

In the paper [22], we consider binomial ideals associated with distriburive lattices. Given a distributive lattice L, in the polynomial ring $K[L]$ we consider the binomial ideal

I_L called join-meet ideal which is generated by the binomials of the form $ab-(a\vee b)(a\wedge b)$, where a and b are incomparable elements in L. These ideals are also called Hibi ideals since they are the defining ideals of the Hibi rings. These rings were introduced and studied by Hibi in a series of papers [31], [32], [33]. They have connections to the representation theory and some other domains [36]. The works [1], [27], [49] study Gröbner bases of Hibi ideals with respect to various monomial orders.

The paper [22] approaches for the first time in literature the study of syzygies of Hibi ideals. For a distributive planar lattice L, it is shown that the regularity of the associated Hibi ideal can be expressed in terms of the combinatorics of the lattice. For an arbitrary non-planar lattice L, bounds for the regularity of I_L are obtained. More precisely, it is shown that the regularity of I_L is greater than or equal to the number of join-irreducible elements of L which are pairwise incomparable minus 1 and smaller than or equal to the number of join-irreducible elements of L minus 1. As an application, it is proved that I_L has a linear resolution if and only if the lattice is isomorphic to the divisor lattice of $2\cdot 3^a$, with $a\geq 1$.

In the paper [35], we introduce an algorithm for computing the Hilbert depth of a finitely generated multigraded module M over the standard multigraded polynomial ring $R=K[X_1,\dots,X_n]$. The algorithm is based on the method presented in [34] and some extra improvements. It may also be adapted for computing the Stanley depth of M if $\dim_K M_a\leq 1$ for all $a\in\mathbb{Z}^n$. Further, we provide an experimental implementation of the algorithm in CoCoA [11] and we use it to find interesting examples. As a consequence, we give complete answers to the following open problems proposed by Herzog in [26]:

1.1. [26, Problem 1.66] *Find an algorithm to compute the Stanley depth for finitely generated multigraded R-modules M with* $\dim_K M_a\leq 1$ *for all* $a\in\mathbb{Z}^n$.

1.2. [26, Problem 1.67] *Let M and N be finitely generated multigraded R-modules. Then*

$$\operatorname{sdepth}(M\oplus N)\geq \operatorname{Min}\{\operatorname{sdepth}(M),\operatorname{sdepth}(N)\}.$$

Do we have equality?

1.3. [26, Text following Problem 1.67] *In the particular case that* $I\subset R$ *is a monomial ideal, does* $\operatorname{sdepth}(R\oplus I)=\operatorname{sdepth} I$ *hold?*

In the papers [45, 47, 48] the authors present new cases when Stanley's conjecture holds true. For all the three papers aforementioned the general setup is the following. Let $I\supsetneq J$ be two squarefree monomial ideals from S. The authors make the extra assumption that I is generated by squarefree monomials of degree $\geq d$, where d is a positive integer. In addition, via a multigraded isomorphism J may be chosen as (0) or generated in degree $\geq d+1$. Let r be the number of monomials of degree d from I and B (resp. C) be the set of squarefree monomials of degree $d+1$ (resp. $d+2$) from $I\setminus J$, where $s=|B|$ and $q=|C|$. In the paper [47], the authors consider the following particular case: I is generated by a monomial f of degree d and by a set E of squarefree monomials of degree $\geq d+1$. In this case they prove that if $s \neq q+1$ and $\operatorname{sdepth}_S(I/J) \leq d+1$ then $\operatorname{depth}_S(I/J) \leq d+1$. In the paper [48] the authors generalize their previous result in the following way: I is minimally generated by squarefree monomials $f_1, \ldots, f_r$ of degree d and a set E consisting of squarefree monomials of degree $\geq d+1$. In this more general situation they prove in any of the cases: 1) $d = 1$, $E = \emptyset$; 2) $r = 1$; 3) $1 < r \leq 3$, $E = \emptyset$ if $\operatorname{sdepth}_S(I/J) \leq d+1$ that $\operatorname{depth}_S(I/J) \leq d+1$. In particular, one can easily see that the main reslut from [47] is recovered by case 2). Starting from this idea, in [45], the author proposes the following a weaker version of Stanley's conjecture:

Conjectura 1.4. *Let $I \subset S$ be an ideal minimally generated by squarefree monomials $f_1, \ldots, f_r$ of degree d and a set E consisting of squarefree monomials of degree $\geq d+1$. If* $\operatorname{sdepth}_S(I/J) \leq d+1$ *then* $\operatorname{depth}_S(I/J) \leq d+1$.

The main result of the paper [45] is a positive answer of this conjecture in any of the following 2 situations: 1) $r \leq 4$, 2) $r = 5$, $E = \emptyset$ and exists $c \in C$ such that $\operatorname{supp}(c) \not\subset \cup_{i=1}^{5} \operatorname{supp}(f_i)$, generalizing in this way the results obtained in [47, 48].

In order to describe the main results obtained in [8], we need to introduce the following notation. Let $\mathscr{A} = \{n_1, n_2, n_3\}$ be a set of positive integers such that $\gcd(n_1, n_2, n_3) = 1$. The toric ideal $I_{\mathscr{A}} \subset K[x_1, x_2, x_3]$ of this configuration of vectors has been studied for the first time by Herzog in [25], who proved that $I_{\mathscr{A}}$ is either complete intersection (in which case is minimally generated by 2 binomials) or almost complete intersection (in which case is minimally generated by 3 binomials). Toric ideals are binomial ideals, whose connection with algebraic statistics was for the first time studied in the seminal paper of Diaconis and Sturmfels [15], generating ever since an ongoing research activity in this field. Of a particular importance in algebraic statistics are the following invariants: Markov complexity $m(\mathscr{A})$ and Graver complexity $g(\mathscr{A})$ (see [55] for details) in the case when $\mathscr{A}$ is a finite set of vectors $\mathbf{a}_1, \ldots, \mathbf{a}_r$ from $\mathbb{N}^n$, where $r \geq 3$ and $n \geq 1$. The fundamental result in this direction is that $g(\mathscr{A}) < \infty$ and a formula of computing it, given in [55, Theorem 3]. For the Markov complexity, which is much more important from the point of view of applications in algebraic statistics, it is only known that is bounded above by the Graver complexity (see [55]). However, a formula for computing it is still unknown. Moreover, the Markov complexity is known only in a few particular cases. Santos and Sturmfels leave as an open question in [55] the computation of the Markov complexity $m(\mathscr{A})$ in the case when $\mathscr{A} = \{n_1, n_2, n_3\}$ in terms of n_1, n_2, n_3, and conjecture that $g(\mathscr{A}) = n_1 + n_2 + n_3$ if $\gcd(n_1, n_2) = \gcd(n_1, n_3) = \gcd(n_2, n_3) = 1$. Unexpectedly, in the paper [8], the authors prove as main results that $m(\mathscr{A}) = 3$ if $I_{\mathscr{A}}$ is almost complete intersection, while $m(\mathscr{A}) = 2$ if $I_{\mathscr{A}}$ is complete intersection. In addition, the authors prove that the conjectured value for $g(\mathscr{A})$ turns out to be wrong. However, the following inequality $g(\mathscr{A}) \geq n_1 + n_2 + n_3$ holds in general. The more surprising the results are since the Graver complexity may be as large as possible, while the Markov complexity is at most 3. The proofs given by the authors rely on the general results obtained in [6, 7] as well as on the precise description of all possible sets of minimal generators given by Herzog in [25].

In the paper [30] the authors study the monomial ideals which can be written as intersection of powers of monomial prime ideals, which they call *monomial ideals of intersection type*. It is a known fact that every squarefree monomial ideal is of intersection type, being the irredundant intersection of its minimal monomial prime ideals. Obviously, among the non-radical monomial ideals, the closest to the squarefree monomial ideals are the monomial ideals of intersection type. Indeed, similar monomial ideals were studied before as well as the defining ideals of tetrahedral curves, see [38, 24]. The authors succeed to characterize in [30, Theorem 1.1], all the monomial ideals $I \subset S$ which are of intersection type. More precisely, I is of intersection type if and only if for every associated prime $\mathfrak{p}$ of I the minimal degree of a generator of the monomial localization $I(\mathfrak{p})$ of I is greater than or equal to the maximal degree of a nonzero socle element of $S(\mathfrak{p})/I(\mathfrak{p})$. In addition, the authors also prove that if I is of intersection type, its presentation as intersection of powers of monomial prime ideals is unique. The exponents of the powers of the monomial prime ideals associated to I, in the case of intersection type ideals are bounded above by $\operatorname{reg}(I(\mathfrak{p}))$, for any associated prime $\mathfrak{p}$ of I, see [30, Theorem 1.3]. In the case when the exponents are equal to the upper bound for every associated prime $\mathfrak{p}$, then the monomial ideal is called of *strong intersection type* and it is proved that these ideals are exactly those monomial ideals I, for which their monomial localizations $I(\mathfrak{p})$ have linear resolution for each $\mathfrak{p} \in \operatorname{Ass}(S/I)$. One section is dedicated to the general properties of monomial ideals of intersection type. It is proved that they are integrally closed, and the support hyperplanes of the Newton polyhedron of such an ideal can be described in terms of the unique irredundant primary decomposition described above. The class of such ideals contains (as proved by the authors) the polymatroidal ideals and the principal Borel ideals (which are the only Borel type ideals with such property). Another important result is the classification of all edge ideals whose second power is of intersection type. An important consequence of this result is the fact that the graphs with the property that the second power of its edge ideal is not of intersection type, have none of the powers of its edge ideal of intersection type.

2. Dissemination of results

We present year by year the way how the results of our team were communicated to the scientific community.

2012 Dorin Popescu, Viviana Ene, Bogdan Ichim, Dumitru Stamate and Mircea Cimpoeas were the organizers of the 20-th edition of the traditional National School on Algebra, "Discrete invariants in commutative algebra and in algebraic geometry", organized in Mangalia in the period 02.09.2012-08.09.2012, see http://math.univ-ovidius.ro/sna/edition.aspx?itemID=6.

This edition benefited of an impressive international presence (11 invited speakers from abroad) and a strong scientifical impact. Also, Viviana Ene, Bogdan Ichim and Dumitru Stamate have participated with a contributed talk to this school. In addition, Bogdan Ichim gave the talk ”Introduction to Normaliz” at Rostock University on 09-05-2012, and another one entitled ”How to compute the multigraded Hilbert depth of a module” at Osnabruck University on 20-11-2012. Also Dorin Popescu gave the talk ”Contributions and new results on Stanley's conjecture” at Kaiserslautern University in July 2013. Another member of the grant, Viviana Ene, had a reserach stage abroad at Essen University, in the framework of a scientific cooperation with Prof. Jurgen Herzog. Andrei Zarojanu participated to the conference ”Workshop for young researchers in Mathematics”, which took place in the period 10.05.2012-11.05.2012 at Constanta, with the talk ”Stanley conjecture on intersection of three monomial primary ideals”.

2013 This year the members of our research team have participated with contributed talks to the following national/international conferences:

1. *Experimental and Theoretical Methods in Algebra, Geometry and Topology*, international conference, Eforie Nord, 21-24 June 2013. The conference benefited of participants from 14 countries, see
   http://math.univ-ovidius.ro/Conference/ETMAGT60/
   Dorin Popescu gave at this conference the talk *A hope for Stanley Conjecture on monomial ideals*.
2. *Joint International Meeting of the American Mathematical Society and the Romanian Mathematical Society*, organized at University ”1 Decembrie 1918” from Alba Iulia, 27-30 June 2013.
   http://imar.ro/ams-ro2013/description.php
   Viviana Ene gave at this conference the talk *Binomial ideals and graphs* .
3. The anniversary conference *Faculty of Sciences - 150 years* which took place in the period 29.08-01.09.2013, at the University of Bucharest.
   http://fmi.unibuc.ro/FMI-150/
   Dorin Popescu gave the talk *Around Stanley's conjecture on monomial ideals*, while Viviana Ene spoke about *Binomial edge ideals*.

4 The 21-st edition of the National School on Algebra, ”Algebraic Methods in Combinatorics”, organized at IMAR, in the period 2-6 September 2013, see
   http://math.univ-ovidius.ro/sna/edition.aspx?itemID=7.
   At this school participated a big number of master students and Ph. D. students from Romania, as well as from abroad.
   Some members of the grant gave the following talks (some of them accompanied also by tutorials):
   – *Algebraic and homological properties of binomial edge ideals* (2 lectures), Viviana Ene,

– *Tools of Combinatorial Commutative Algebra*, Dumitru Stamate
– *Matroids and realisability*, Dumitru Stamate
– *Polymatroidal ideals* (2 lectures), Marius Vladoiu

There were also presented the following short contribution talks:
– *Stanley depth of quotient of monomial complete intersection ideals*, Mircea Cimpoeas
– *Depth of some special monomial ideals*, Andrei Zarojanu.

In addition all the members of this grant gave regularly talks about their recent results at the scientific seminar of Commutative Algebra and Combinatorics ”Nicolae Radu”, organized weekly by IMAR and hosted by the Faculty of Mathematics and Computer Science. An archive of this scientific seminar with the talks given since October 2010 can be consulted here

`http://www.imar.ro/organization/activities/archive/seminars_arh_sem_19_s.php`.

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