SCIENTIFIC REPORT OF THE GRANT
PN–II–ID–PCE–2011–3–1023, NR. 247/2011,
FOR THE PERIOD 01.01.2014 – 30.11.2014

INTRODUCTION

We survey here the activities and the results obtained by the members of the grant PN–II–ID–PCE–2011–3–1023, nr. 247/2011, titled ”Algorithmic and theoretical methods for studying monomial and binomial ideals with applications in combinatorics, commutative algebra and graph theory”, principal investigator Prof. dr. Dorin Popescu, during 01.01.2014 – 30.11.2014. Section 1 sums up the scientific results obtained in this timeframe, and Section 2 describes how these results were disseminated, the research stages abroad and the participation to various conferences. All these results and information is available online at

`https://dl.dropboxusercontent.com/u/112281424/grantPCE-2011-3-1023/indexDP.html`

For 2014 we may list the following achievements:

Published papers:

(1) D. Popescu, *Depth of factors of squarefree monomial ideals,* **Proceedings AMS** 142, 1965–1972, 2014.
(2) H. Charalambous, A. Thoma, M. Vladoiu, *Markov complexity of monomial curves,* **J. Algebra** 417, 391–411, 2014.
(3) J. Herzog, M. Vladoiu, *Monomial ideals with primary components given by powers of monomial prime ideals,* **Electronic J. Combinatorics** 21(1), #P1.69, 2014.
(4) B. Ichim, A. Zarojanu, *An algorithm for computing the multigraded Hilbert depth of a module,* **Experimental Mathematics** 23(3), 322–331, 2014.
(5) V. Ene, J. Herzog, T. Hibi, *Koszul binomial edge ideals* , Bridging Algebra, Geometry, and Topology, **Springer Proceedings in Mathematics & Statistics**, 96, D. Ibadula, W. Veys (Eds.) Springer, 127–138, 2014.
(6) M. Cimpoeaş, *Stanley depth of quotient of monomial complete intersection ideals,* **Comm. in Algebra** 42, 4274–4280 (2014).

Papers accepted for publication:

(1) V. Ene, J. Herzog, T. Hibi, *Linear flags and Koszul filtrations*, to appear in **Kyoto J. Math**.
(2) V. Ene, J. Herzog, T. Hibi, *Linearly related polyominoes*, to appear in **J. Algebraic Combinatorics**, DOI 10.1007/s10801-014-0560-3.
(3) F. Chaudhry, A. Dokuyucu, V. Ene, *Binomial edge ideals and rational normal scrolls*, to appear in **Bull. Iranian Math. Soc.**

(4) V. Ene, *Syzygies of Hibi rings*, to appear in **Acta Mathematica Vietnamica**, special volume dedicated to the 60th birthday of Professor N. V. Trung.
(5) V. Ene, A. Zarojanu, *On the regularity of binomial edge ideals*, to appear in **Mathematische Nachrichten**, DOI: 10.1002/mana.201300186.
(6) D. Popescu, A. Zarojanu, Three generated squarefree monomial ideals, to appear in **Bull. Math. Soc. Sci. Math. Roumanie**, 58(106), no. 3, 2015, see arXiv:1307.8292.

Submitted papers:

(1) V. Ene, J. Herzog, S. Saeedi Madani, *A note on the regularity of the Hibi rings*, preprint, arXiv:1404.2554.
(2) V. Ene, J. Herzog, T. Hibi, S. Saeedi Madani, *Pseudo-Gorenstein and level Hibi rings*, preprint, arXiv:1405.6963.
(3) A. Dimca, D. Popescu, *Hilbert series and Lefschetz properties of dimension one almost complete intersections*, preprint, arXiv:1403.5921.
(4) D. Popescu, *Depth in a pathological case*, preprint, arXiv:1406.1398.

1. Scientific results obtained in 2014

We next outline the results obtained by the team members in the 8 papers written in 2014.

1. V. Ene, J. Herzog, T. Hibi, *Linear flags and Koszul filtrations*, to appear in Kyoto J. Math.

Let K be a field, $S = K[x_1,\dots,x_n]$ a polynomial ring and $I \subset (x_1,\dots,x_n)^2$ a graded ideal in S. We consider the K-algebra $R = S/I$ and we let $\mathfrak{m}$ be its maximal graded ideal. The algebra R is called Koszul if $K = R/\mathfrak{m}$ has an R-linear resolution. One knows that if R is Koszul, then I is generated by quadrics, and that if I has a quadratic Groebner basis, then R is Koszul. In [26] the authors introduced the concept of strongly Koszul algebra. It is proven that these form a proper subclass of the Koszul ones.

Another sufficient condition for Koszulity is the existence of a Koszul filtration for R, see [8, 7].

In this paper one studies the connection among the following properties:

(i) R has a Koszul filtration;
(ii) I has a quadratic Groebner basis.

In Theorem 1.1 one shows that if I has a quadratic Groebner basis with respect to revlex, then R has a linear "flag". A chain of ideals in R generated by linear forms

$$(0) = I_0 \subset I_1 \subset \cdots \subset I_n = (\bar{x}_1,\dots,\bar{x}_n)$$

is called a linear flag if for any j, I_{j+1}/I_j is a cyclic S-module whose annihilator is generated by linear forms.

In general, even when I is a binomial ideal with a quadratic Groebner basis with respect to revlex, the quotients $(\bar{x}_{i+1},\dots,\bar{x}_n) : \bar{x}_i$ may not be generated by subsets of $\{\bar{x}_1,\dots,\bar{x}_n\}$. However, for some classes of biomial ideals coming from combinatorics, the latter property holds, see Theorem 1.6, Theorem 2.1, Corollary 2.6.

2. V. Ene, J. Herzog, T. Hibi, *Linearly related polyominoes*, to appear in J. Algebraic Combin. DOI 10.1007/s10801-014-0560-3.

The polyomino ideals were introduced in [43]. This class includes the two-sided ladder determinantal ideals and the Hibi binomial ideals associated to planar distributive lattices. A polyomino ideal is generated by a collection of 2-minors of a generic matrix X of type $m \times n$. In this paper we also considered such collections of minors associated to a convex configuration.

It is a difficult question to understand the resolution of such ideals. The regularity of Hibi rings associated to distributive lattices is computed in [17]. Sharpe [S1,S2] had proved that the determinantal ideal $I_2(X)$ generated by the 2-minors of the generic matrix X has linear relations. Kurano [31] extended this result to ideals of the form $I_t(X)$ where $2 \leq t \leq \min(m,n)$. Hashimoto proved that, in general, the resolution of such ideals depends on the characteristic of the basefield, see [19]. However, Bruns and Herzog proved in [3] that the second Betti number $\beta_2(I_2(X))$ does not depend on $char(K)$.

Using the square-free divisor complex technique introduced by Bruns si Herzog in [3], we classified the polyomino ideals with linear relations. Moreover, we determined necessary conditions for the polynomino ideal $I_{\mathscr{P}}$ associated to a polyomino $\mathscr{P}$ to have a linear resolution. In particular, we were able to derive new results concerning the resolution of the Hibi ideals associated to distributive lattices.

3. F. Chaudhry, A. Dokuyucu, V. Ene, *Binomial edge ideals and rational normal scrolls*, to appear in Bull. Iranian Math. Soc.

Given a Hankel matrix of type $2 \times n$,

$$X = \begin{pmatrix} x_1 & \dots & x_{n-1} & x_n \\ x_2 & \dots & x_n & x_{n+1} \end{pmatrix}$$

and a graph G with vertex set $[n]$, we consider the ideal $I_G \subset R = K[x_1,\dots,x_{n+1}]$ generated by the maximal minors of the matrix X corresponding to the edges of G. For a closed graph endowed with a natural labeling of its vertices, the generators of I_G form also a Groebner basis with respect to revlex. As a consequence, the ideal I_G is Cohen-Macaulay of dimension $1+$ the number of connected components of G.

In Section 2 we determine the associated primes of I_G when G is connected and closed. This results allows to characterize the closed and connected graphs such that I_G is a radical ideal. Working under the same hypotheses on G, one gets that I_G is Cohen-Macaulay and a set-theoretic complete-intersection. In the last section of the paper we prove that when G is a closed graph, the regularity of R/I_G is at most the number of maximal cliques in G. As a corollary, one characterizes the closed graphs G such that I_G has a linear resolution.

4. V. Ene, *Syzygies of Hibi rings*, to appear in Acta Mathematica Vietnamica, special volume dedicated to the 60th birthday of Professor N. V. Trung.

This is a survey on recent results concerning the resolutions of (generalized) Hibi rings associated to distributive lattices. This paper is structured in 3 sections: Hibi rings and their Gröbner bases, Level and pseudo-Gorenstein Hibi rings, The regularity of Hibi rings.

Hibi rings have been introduced by Hibi in 1987 [27]. They occur in various algebraic and combinatorial contexts.

Let L be a finite distributive lattice. A famous theorem of Birkhoff says that L is isomorphic to the lattice $\mathscr{I}(P)$ of order ideals of the poset P consisting of the join-irreducible elements of L. Let $P = \{p_1, \ldots, p_n\}$ and $R = K[t, x_1, \ldots, x_n]$ the polynomial ring in $n+1$ indeterminated over a field K. The Hibi ring $R[L]$ associated to L is the toric ring over K generated by the monomials $u_\alpha = t\prod_{p_i \in \alpha} x_i$, where α is any order ideal P. The ring $R[L]$ is a standard graded K-algebra with $\deg u_\alpha = 1$ for all $\alpha \in L$. The defining ideal of this K-algebra is called the Hibi ring associated to the lattice L.

Hibi proved that such rings are ASL (algebras with straightening laws), normal Cohen-Macaulay domains. Moreover, he characterized in terms of P when is $R[L]$ a Gorenstein ring. Several properties of Hibi rings have been recently under scrutiny. E.g., Groebner bases for the defining ideal in [1, 25, 44], various Koszul-related properties in [6, 26], the divisor class ring of a Hibi ring in [20], geometric properties of the variety associated to the Hibi ideal, see [50]. Most of these results are included for the first time in a survey.

In [15] generalized Hibi rings and ideals were introduced. In the first section of our paper we show that these also have the structure of an ASL and we also obtain a Groebner basis for the defining ideal.

In the second section of the paper we present recent results concerning level or pseudo-Gorenstein (generalized) Hibi rings. Some of these originated in [V. Ene, J. Herzog, T. Hibi, S. Saeedi Madani, *Pseudo-Gorenstein and level Hibi rings*, arXiv:1405.6963.]

In the last section of the survey we give a formula for the regularity of $R[L]$ in terms of the poset P. This formula was first obtained in [V. Ene, J. Herzog, S. Saeedi Madani, *A note on the regularity of Hibi rings*, arXiv:1404.2554]. One also derives characterizations of the lattices L producing Hibi rings $R[L]$ with a linear or pure minimal resolution.

5. V. Ene, J. Herzog, S. Saeedi Madani, *A note on the regularity of the Hibi rings*, arXiv:1404.2554.

Let $R[L]$ be the Hibi ring associated to a distributive lattice $L = \mathscr{I}(P)$, where P is a finite poset. One already knows from the work of Hibi that $\dim R[L] = |P| + 1$, hence $\operatorname{proj}\dim R[L] = |L| - |P| - 1$ since $R[L]$ is Cohen-Macaulay. A first attempt to study the regularity of $R[L]$ was made in [17]. It is proven there that when P is a planar lattice, the regularity of $R[L]$ equals the maximum number of squares in a cyclic sublattice of L. In the current paper we prove that for any distributive lattice L one has

$$\operatorname{reg} R[L] = |P| - \operatorname{rank} P - 1.$$

For the proof we use the combinatorial description given by Hibi in [27] for the generators of the canonical ideal of $R[L]$. We also give the ideas for a purely combinatorial proof of this formula. This allows us to characterize lattices L such that the regularity of $R[L]$ is 1 or 2.

6. V. Ene, J. Herzog, T. Hibi, S. Saeedi Madani, *Pseudo-Gorenstein and level Hibi rings*, arXiv:1405.6963.

This paper continues the study of Hibi rings and ideals, continuing the work in the preceding paper. For a Cohen-Macaulay standard graded algebra R with canonical module ω_R, one knows that R is Gorenstein if and only if ω_R is cyclic. This condition on ω_R may be relaxed in at least two directions. When ω_R is generated in a single degre, R is called level. When there is a single generator for ω_R lying in the smallest degree, we call R pseudo-Gorenstein. The latter concept is introduced here for the first time.

In this paper we characterize in terms of the poset P of join-irreducible elements, the distributive lattices $L = \mathscr{I}(P)$ such that the corresponding Hibi ring is pseudo-Gorenstein. Moreover, we explain the connections to with the level or Gorenstein algebras. We introduce the concept of hyper-planar lattice, which extends naturally the notion of planar lattice. In the paper we prove that a hyper-planar lattice is pseudo-Gorenstein (i.e. its Hibi ring has this property) if and only if all the chain in a canonical decomposition of P have the same length.

A sufficent condition for $R[L]$ to be level has appeared in [34]. This is not usually necessary. In our paper we present a necessary condition for $R[L]$ to be level and we conjecture that this is also sufficient. This condition is expressed in terms of the combinatorics of the poset P. For planar lattices satisfying a certain regularity condition we prove that the above mentioned necessary condition is also enough to guarantee the level property.

In the last section of the paper we study the level and pseudo-Gorenstein properties for generalized Hibi rings.

7. A. Dimca, D. Popescu, *Hilbert series and Lefschetz properties of dimension one almost complete intersections*, arXiv:1403.5921.

Let $S = K[x_0,\dots,x_n]$ be a polynomial ring over a field K of zero characteristic and $f = f_0,\dots,f_n$ an almost complete intersection system of polynomials with deg $f_i = d_i$. We describe the Hilbert series of $S/(f)$ in terms of the Hilbert series of S/I, where I is the saturation of (f) in S. When I is a complete intersection, one can fully determine the Hilbert series of $S/(f)$.

This problem is motivated by singularity theory. Let $V \subset \mathbf{P}^n$ be a projective hypersurface with isolated singularities given by an equation $g = 0$, $g \in S$. Then the partial derivatives $g_0,\dots,g_n$ satisfy the above conditions, hence one can describe the Hilbert series of the Milnor algebra $M(g) = S/(g_0,\dots,g_n)$ associated to g. This may be used to compute various invariants of V.

In this context one studies the Lefschetz type properties of the algebra $M(g)$. The Lefshetz properties have been studied for quite a while. Nevertheless, several conjectures are still open, even in the case when $f_0,\dots,f_n$ is a regular sequence. What is known works for $n \leq 2$. Other several small results are proved for $n > 2$ and this topic seems to be generously funded by the NSA. It is believed that they may be useful in criptography. In the current paper we describe a Lefschetz type property when f is an almost complete intersection and $n = 2$, in particular for $M(g)$ in the case when $n = 2$. Counterexamples are available when $n > 2$.

8. D. Popescu, *Depth in a pathological case*, arXiv:1406.1398.

Let $S = K[x_1,\dots,x_n]$ be a polynomial algebra over the field K and let $J \subset I \subset S$ be two monomial ideals. We assume I is minimally generated by some squarefree monomials $f_1,\dots,f_r$ of degree d and a set of E of squarefree monomials of degree $\geq d+1$, and J is either zero or it is generated by squarefree monomials of degree $\geq d+1$. Let w_{ij} be the least common multiple of f_i, f_j,

$1 \leq i < j \leq r$ and W the set of all possible w_{ij}. We denote by B the set of squarefree monomials of degree $d+1$ in $I \setminus J$.

In this paper we show that if $B \setminus E \subset W$ (in such a situation we call I/J a pathological case), then $\mathrm{depth}_S I/J \leq d+1$. Therefore Stanley's conjecture holds for the pathological case, since the paper proc3 already shows that if the so called Stanley depth of I/J equals d, then also $\mathrm{depth}_S I/J = d$. For a proof we first reduce to the case $E = \emptyset$. This follows from the so called Depth Lema. When $E = \emptyset$ we make an interesting observation: if $r > 1$ and $\mathrm{depth}_S I/(J, f_r) = d$, then $\mathrm{depth}_S I/J \leq d+1$. One uses here the Koszul homology. The proofs are difficult and they are inspired by the study of several examples obtained with the computer algebra system SINGULAR.

2. Dissemination of results

The members of the research team presented the following talks at the 2014 edition of the National School on Algebra that took part at IMAR, during September 1-5.
(see `http://math.univ-ovidius.ro/sna/edition.aspx?cat=GeneralInfo&itemID=8`):

(1) Viviana Ene, ”Hibi rings and their Grobner bases”, 1.09.2014.
(2) Miruna Roşca, ”Vertex cover algebras of weighted graphs”, 1.09.2014.
(3) Viviana Ene, ”Level and pseudo - Gorenstein Hibi rings”, 2.09.2014.
(4) Andrei Zarojanu, ”An algorithm for computing the multigraded Hilbert depth of a module”, 2.09.2014.
(5) Viviana Ene, ”The regularity of Hibi rings”, 4.09.2014.

We would also like to mention that some of the team members were among the organizers of this edition:Viviana Ene, Miruna Roşca, Andrei Zarojanu, Dumitru Stamate and Marius Vlădoiu. The recent results are regularly presented at the weekly Commutative Algebra Seminar run jointly by IMAR and the University of Bucharest, see the archive of the talks at
`http://www.imar.ro/organization/activities/archive/seminars_arh_sem_19_2014_s.php`

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