SCIENTIFIC REPORT OF THE GRANT PN–II–ID–PCE–2011–3–1023, NR. 247/2011, FOR THE PERIOD 01.12.2014 – 31.12.2015

Introduction

We survey here the activities and the results obtained during the period 01.12.2014 – 31.12.2015 by the members of the grant PN–II–ID–PCE–2011–3–1023, nr. 247/2011, entitled "Algorithmic and theoretical methods for studying monomial and binomial ideals with applications in combinatorics, commutative algebra and graph theory", whose principal investigator is Prof. dr. Dorin Popescu. Section 1 sums up the scientific results obtained in this time frame, and Section 2 describes how these results were disseminated, the research stages abroad and the participation to various conferences. All these results and information are available online at

`https://dl.dropboxusercontent.com/u/112281424/grantPCE-2011-3-1023/indexDP.html`

The scientific results obtained during the above mentioned period are listed below:

Published papers:

(1) V. Ene, J. Herzog, S. Saeedi Madani, *A note on the regularity of Hibi rings*, **Manuscripta Math.** 148(3), 501-506, 2015.
(2) V. Ene, J. Herzog, T. Hibi, *Linearly related polyominoes*, **J. Algebraic Combinatorics** 41 (4), 949-968, 2015.
(3) V. Ene, J. Herzog, T. Hibi, S. Saeedi Madani, *Pseudo-Gorenstein and level Hibi rings*, **J. Algebra** 431, 138-161, 2015.
(4) V. Ene, J. Herzog, T. Hibi, *Linear flags and Koszul filtrations*, **Kyoto J. Math.** 55(3), 517-530, 2015.
(5) F. Chaudhry, A. Dokuyucu, V. Ene, *Binomial edge ideals and rational normal scrolls*, **Bull. Iranian Math. Soc.** 41, no. 4, 971-979, 2015.
(6) V. Ene, A. Zarojanu, *On the regularity of binomial edge ideals*, **Math. Nachrichten** 288(1), 19-24, 2015.
(7) V. Ene, *Syzygies of Hibi rings*, **Acta Mathematica Vietnamica**, Special Issue on: Commutative Algebra and its Interaction with Algebraic Geometry and Combinatorics II 40(3), 403-446, 2015.
(8) D. Popescu, A. Zarojanu, *Three generated, squarefree, monomial ideals*, **Bull. Math. Soc. Sci. Math. Roumanie**, 58(106), no 3, 359-368, 2015.

Papers accepted for publication:

(1) D. Popescu, *Stanley depth on five generated, squarefree, monomial ideals*, **Bull. Math. Soc. Sci. Math. Roumanie**, 59(107), no 1, 2016, arXiv:AC/1312.0923v5.
(2) D. Popescu, *Depth in a pathological case*, to appear in **Bull. Math. Soc. Sci. Math. Roumanie**, arXiv:AC/1406.1398v6.
(3) A. Dimca, D. Popescu, *Hilbert series and Lefschetz properties of dimension one almost complete intersections*, to appear in **Communications in Algebra**, arXiv:AG/14035921v2.

Papers written in 2015 and submitted for publication:

(1) D. Popescu, *Around General Neron Desingularization*, 2015, arXiv:AC/1504.06938.
(2) A. Popescu, D. Popescu, *A method to compute the General Neron Desingularization in the frame of one dimensional local domains*, 2015, arXiv:AC/1508.05511.
(3) D. Popescu, *Artin approximation property and the General Neron Desingularization*, 2015, arXiv:AC/1511.06967.
(4) M. Cimpoeas, D. Stamate *On intersections of complete intersection ideals*, preprint 2015.
(5) M. Cimpoeas, *Stanley depth of the path ideal associated to a line graph*, 2015, arXiv.1508.07540.
(6) M. Cimpoeas, *On the quasi-depth of squarefree monomial ideals and the sdepth of the monomial ideal of independent sets of a graph*, 2015, arXiv.1511.06974v1.

1. Description of the scientific results obtained during 2015

Next we briefly summarize the most important results obtained by the team members in the 6 papers written in 2015.

1. D. Popescu, *Around General Neron Desingularization*, arXiv:AC/1504.06938.

An old result of us is the so called the General Neron Desingularization theorem (shortly GND), which says that given a regular morphism of Noetherian rings $A \to A'$, an A-algebra of finite type B then any A-morphism $v : B \to A'$ factors through a smooth A-algebra C, that is v is a composite A-morphism $B \to C \to A'$. This theorem extends the Neron Desingularization given in the frame of DVR. The first example of GND in dim > 1, given for the convergent power series rings over $\mathbf{C}$, is the Ploski's theorem.

We extend the Ploski's theorem in the frame of algebraic power series rings over excellent Henselian local rings. An easy proof of GND over DVR is included. We note that if (A,m) is a DVR and A' is its completion then C could be chosen to be the same for any A-morphism $v' : B \to A'$ such that $v' \equiv v$ modulo $m^{2c+1}A'$ for some $c \in \mathbf{N}$ depending on B. Moreover, the set of these A-morphisms is in bijection with A'^s for some $s \in \mathbf{N}$.

Many years ago we showed that GND implies a partial positive solution for the Bass-Quillen Conjecture. With the help of a theorem of Vorst we prove here that given a regular local ring (R,m,k) such that either R contains a field, or the characteristic of k is not in m^2 then any finitely generated projective module over a ring $R[x]/I$, $x = (x_1,\ldots,x_n)$ is free providing I is a monomial ideal.

2. A. Popescu, D. Popescu, *A method to compute the General Neron Desingularization in the frame of one dimensional local domains*, arXiv:AC/1508.05511.

The main purpose of this paper is to give an algorithmic method to compute the General Neron Desingularization in the frame of regular morphisms $A \to A'$ of Noetherian local domains of dimension one. Here (A,m,k) is a local essentially of finite type $\mathbf{Q}$ and (A',mA',k') is a local subring of a complete local $\mathbf{C}$-algebra. For simplicity, we suppose that $k \subset A$, $k' \subset A'$, otherwise the computations are very difficult. However, even in this case the illustration from the last example has an output given by the Computer Algebra System SINGULAR which is reasonable to present only modulo a high enough power of m.

Another result of this paper is an extension of Greenberg'strong approximation theorem, which says that given an excellent Henselian DVR (A,m) and a system of polynomials $f = (f_1,\dots,f_s)$ from $A[Y]$, $Y = (Y_1,\dots,Y_n)$ there exists a linear map $\nu : \mathbf{N} \to \mathbf{N}$ with the following property:

"If $y \in A^n$ satisfies $f(y) \equiv 0$ modulo $m^{\nu(c)}$ for some $c \in \mathbf{N}$ then there exists a solution $y' \in A^n$ such that $y' \equiv y$ modulo m^c."

We extend Greenberg's result in the frame of local excellent Henselian Cohen-Macaulay rings of dimension one.

3. D. Popescu, *Artin approximation property and the General Neron Desingularization*, arXiv:AC/1511.06967.

This paper is a survey for the "Contributions to the Eighth Congress of Romanian Mathematicians", 2015. It contains some results obtained by us this year but also others obtained many years ago. An important part concerns the so-called the Artin approximation in nested subring condition. More precisely, we have the following result:

Theorem. Let K be a field, $A = K\langle x\rangle$, $x = (x_1,\dots,x_m)$, $f = (f_1,\dots,f_r) \in K\langle x,Y\rangle^r$, $Y = (Y_1,\dots,Y_n)$ and $0 \le s_1 \le \dots \le s_n \le m$, c be some non-negative integers. Suppose that f has a solution $\hat{y} = (\hat{y}_1,\dots,\hat{y}_n)$ in $K[[x]]$ such that $\hat{y}_i \in K[[x_1,\dots,x_{s_i}]]$ for all $1 \le i \le n$. Then there exists a solution $y = (y_1,\dots,y_n)$ of f in A such that $y_i \in K\langle x_1,\dots,x_{s_i}\rangle$ for all $1 \le i \le n$ and $y \equiv \hat{y}$ modulo $(x)^c K[[x]]$.

Here we present the main ideas of the proof together with a special application to the algebraic versal deformations.

An extension of Greenberg's result (see above) was obtained with A. Popescu in the frame of excellent Henselian local Cohen-Macaulay rings of dimension one. Here we announce that this result is also true for excellent Henselian local rings of dimension one which are not Cohen-Macaulay, the linear map ν being given by $c \to e + c$, where e depends by f and on a reduced primary decomposition of (0) in A. This result is part of a forthcoming paper with G. Pfister.

4. M. Cimpoeas, D. Stamate, *On intersections of complete intersection ideals*, preprint.

Let K be a field and $S = K[x_1,\dots x_r]$ be the polynomial ring in the variables $x_1,\dots,x_r$. An ideal $I \subset S$ is called a *complete intersection* (CI for short) if it is minimally generated by height(I) elements. This is a strong condition which is rarely preserved by taking intersections of such ideals. In this article we exhibit several infinite families of CI toric ideals in S such that any intersection of ideals in the same family is again a CI. For an affine semigroup $H \subset \mathbb{N}$ the semigroup ring $K[H]$ is the subalgebra in $K[t]$ generated by the monomials t^h, $h \in H$.

If $a_1,\ldots,a_r$ generate H minimally, we define the toric ideal I_H as the kernel of the K-algebra map $\varphi : S \to K[H]$ letting $\varphi(x_i) = t^{a_i}$. Consider the list of integers $\mathbf{a} = a_1 < a_2 < \cdots < a_r$. We denote $I(\mathbf{a})$ the kernel of the K-algebra map $\varphi : S \to K[\langle \mathbf{a}\rangle]$ letting $\varphi(x_i) = t^{a_i}$, where we let $\langle \mathbf{a}\rangle$ be the semigroup generated by $a_1,\ldots,a_r$. If they generate $\langle \mathbf{a}\rangle$ minimally, we call $I(\mathbf{a})$ the toric ideal of $\langle \mathbf{a}\rangle$. If k is any integer we let $\mathbf{a}+k = (a_1+k,\ldots,a_r+k)$. The study of properties of the family of ideals $\{I(\mathbf{a}+k)\}_{k\geq 0}$ is a recent topic of interest. We use Gröbner bases techniques to derive new information about intersections of such CI ideals. Our main result, is where we prove that for a fixed $\mathbf{a}$ intersections of CI-ideals $I(\mathbf{a}+k)$ with large enough shifts k produce another CI ideal.

5. M. Cimpoeas, *Stanley depth of the path ideal associated to a line graph*, arXiv.1508.07540.
Let $n \geq m \geq 1$ be two integers, let K be a field and $S = K[x_1,\ldots,x_n]$ the polynomial ring over K. Let $\Delta_{n,m}$ be the simplicial complex with the set of facets $\mathscr{F}(\Delta_{n,m}) = \{\{1,2,\ldots,m\},\{2,3,\ldots,m+1\},\cdots,\{n-m+1,\ldots,n\}\}$. We denote $I_{n,m} = (x_1x_2\cdots x_m, x_2x_3\cdots x_{m+1},\ldots,x_{n-m+1}\cdots x_n)$, the associated facet ideal. Note that $I_{n,m}$ is the path ideal of the line graph L_n, provided with the direction given by $1<2<\ldots<n$. We prove that $\operatorname{sdepth}(S/I_{n,m}) = \operatorname{depth}(S/I_{n,m}) = n+1-\left\lfloor \frac{n+1}{m+1}\right\rfloor - \left\lceil \frac{n+1}{m+1}\right\rceil$. This result, generalize, for example, the case $m=2$, when $I_{n,2}$ is the edge ideal of the line graph L_n.

The proof is technical, and it use induction on $m \geq 1$ and $n \geq m$. I will explain the idea using the following example. If $n=6$ and $m=3$, then $I_{6,3} = (x_1x_2x_3, x_2x_3x_4, x_3x_4x_5, x_4x_5x_6) \subset S := K[x_1,\ldots,x_6]$. Let $L_0 = I_{6,3}$, $L_1 = (L_0 : x_3) = (x_1x_2, x_2x_4, x_4x_5)$ and $J_1 = (L_0, x_3) = (x_3, x_4x_5x_6)$. Since $L_1 \cong I_{4,2}S$, it follows that

$$\operatorname{depth}(S/L_1) = \operatorname{sdepth}(S/L_1) = \operatorname{depth}(S/I_{4,2}S) = 2 + \operatorname{depth}(K[x_1,\ldots,x_4]/I_{4,2}) = 2 + \left\lceil \frac{4}{3} \right\rceil = 4.$$

On the other hand, since J_1 is a complete intersection, $\operatorname{depth}(S/J_1) = \operatorname{sdepth}(S/J_1) = 4$. We consider the short exact sequence $0 \to S/L_1 \to S/L_0 \to S/J_1 \to 0$. It follows that $\operatorname{sdepth}(S/L_0) \geq 4$. On the other hand, since $L_1 = (L_0 : x_3)$, one has $\operatorname{sdepth}(S/L_0) \leq \operatorname{sdepth}(S/L_1) = 4$. Thus $\operatorname{sdepth}(S/L_0) = 4$. Also, $\operatorname{depth}(S/L_0) = 4$. One possible way to generalize our main result, would be to prove that $\operatorname{sdepth}(S/I_{n,m}^k) = \operatorname{depth}(S/I_{n,m}^k)$ for any $k \geq 1$. Furthermore, we might conjecture that if Δ is a simplicial tree, then $\operatorname{sdepth}(S/I(\Delta)^k) = \operatorname{depth}(S/I(\Delta)^k)$ for any $k \geq 1$.

6. M. Cimpoeas, *On the quasi-depth of squarefree monomial ideals and the sdepth of the monomial ideal of independent sets of a graph*, arXiv.1511.06974v1.
Let $n \geq 3$ be an integer, let K be a field and $S = K[x_1,\ldots,x_n]$ the polynomial ring over K. Let $I \subset J$ be two monomial ideals, and let $\mathscr{P} = \mathscr{P}_{J/I} := \{\sigma \subset [n] \,:\, x_\sigma \in J \setminus I\}$, where $x_\sigma = \prod_{i\in\sigma} x_i$. We denote $\mathscr{P}_k = \{A \in \mathscr{P} \,:\, |A| = k\}$ and $\alpha_k = |\mathscr{P}_k|$. Assume that $\operatorname{sdepth}(J/I) = d$. According to a result of Herzog, Vladoiu and Zheng, it follows that $\mathscr{P}$ admits a partition $\mathscr{P} = \bigcup_{i=1}^r [F_i, G_i]$ with $|G_i| \geq d$ for all i. Let $\beta_k = |\{i \,:\, |F_i| = k\}$. One can easily see that $\beta_0 = \alpha_0$ and $\beta_k = \alpha_k - \beta_0\binom{d}{k} - \beta_1\binom{d-1}{k-1} - \cdots - \beta_{k-1}\binom{d-k}{1}$, for all $1 \leq t \leq d$. We may ask, given $\alpha_0,\ldots,\alpha_n$, and constructing β_k as above, which is the largest d for which the β_k's are nonnegative. We denote this number with $\operatorname{qdepth}(J/I)$. Obviously, $\operatorname{sdepth}(J/I) \leq \operatorname{qdepth}(J/I)$.

Of course, $\operatorname{qdepth}(J/I)$ can be very easily calculated with a computer, while $\operatorname{sdepth}(J/I)$ cannot, since the algorithm to generate partitions has exponential time growth. We verify the famous example which disproved the Stanley conjecture, i.e. $\operatorname{sdepth}(S/I) < \operatorname{depth}(S/I)$, showing that, also, $\operatorname{qdepth}(S/I) < \operatorname{depth}(S/I)$. Now, let $I \subset S$ be a squarefree monomial ideal generated in degree $m < n$ with $g = |G(I)|$. We proved that if $\binom{n+m-d-1}{m} < g$, then $\operatorname{qdepth}(S/I) \leq d-1$. Also, in particular, if $\binom{n-1}{m} < g$, then $\operatorname{qdepth}(S/I) = \operatorname{sdepth}(S/I) = m-1$.

Let $G = (V,E)$ be a graph with the vertex set $V = [n]$ and edge set E. A set of vertices S is *independent* if there are no elements i and j of S such that $\{i,j\} \in E$. We denote by $Ind(G)$ the set of all the independent sets of G. Let $T := K[s_i,t_i : i \in [n]]$ be the ring of polynomials in two sets of n variables. For any $S \in Ind(G)$, we consider the monomial $m_S := \prod_{i\in S} s_i \prod_{i\notin S} t_i$. Let $I := (m_S : S \in Ind(G)) \subset T$. I is called the monomial ideal of independent sets associated to graph G. Let $g = |G(I)|$. We proved that $\gamma(G) \geq \operatorname{sdepth}(T/I) \geq \operatorname{depth}(T/I)$, where $\gamma(G) = \max\{d : \binom{3n-d-1}{n} \geq g\}$. We conjectured that $\operatorname{sdepth}(T/I) = \gamma(G)$.

2. Dissemination of results in 2015

The members of the research team had several international/national talks during 2015. Undoubtedly, the most prestigious international exposure this year belongs to the director of this project who was invited speaker for the special semester dedicated to Artin Approximation and Singularity Theory at CIRM, Luminy (France)
(see `http://chairejeanmorlet-1stsemester2015.weebly.com/program-overview.html`)

He was invited to deliver a four lecture series as part of a mini-course for Ph.D students in the period 26-30 January 2015, with the theme "Introduction to Artin Approximation and the Geometry of Power Series Spaces". The titles of the four lectures are: 1) Smooth morphisms, 2) The smooth locus of a morphism, 3) Elkik's theorem, and 4) General Neron Desingularization in dimension 1. Later on, he was invited speaker at the conference "Applications of Artin Approximation in Singularity Theory", held in the period February 2-6, 2015, giving a talk on "Artin approximation, versal deformations, and maximal Cohen-Macaulay modules". Another talk presented by Dorin Popescu was "On the regular local rings" at the conference "Artin Approximation and Infinite Dimensional Geometry" organized in the period 23-27 March 2015 at CIRM, Luminy. His presentations were highly appreciated and consequently he was invited in the period 2.11-10.11.2015 for a research stage at the prestigious university KU Leuven, where he also delivered a talk in the general seminar on the 4th of November.

Every member of our research team has delivered a talk at "The Eighth Congress of Romanian Mathematicians" held in Iaşi, Romania, June 26 - July 1, as a proof of the international research visibility of our team,
(see `http://www.math.uaic.ro/cmr2015/programme/Algebra%20and%20Number%20Theory.pdf`)

In addition, the members of the research team support scientifically, as part of the tradition during the last years, the National School on Algebra. This year, the 23rd edition of the National School on Algebra took place at IMAR, during the period 31.08-04.09. The lectures delivered by the members of our team can be seen at the following
(see `http://math.univ-ovidius.ro/sna/edition.aspx?cat=GeneralInfo&itemID=9`)

We would also like to mention that three of the team members were among the organizers of this edition: Viviana Ene, Dumitru Stamate and Marius Vlădoiu.

Moreover, the recent results are regularly presented at the weekly Commutative Algebra Seminar run jointly by IMAR and the University of Bucharest, see the archive of the talks at http://www.imar.ro/organization/activities/archive/seminars_arh_sem_19_2015_s.php

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Director of the grant,

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