Final Scientific Report

regarding the achievements within this project in the period January 2012 – December 2016

We are going to present in this report the scientific activity carried out through the whole period of this grant, PN–II–ID–PCE–2011–3–1023, nr. 247/2011, having the title "Algorithmic and theoretical methods for studying monomial and binomial ideals with applications in combinatorics, commutative algebra and graph theory" and whose director is Prof. dr. Dorin Popescu. For an optimal description we will present first the scientific articles obtained during this grant, and which have acknowledgments for the financial support of this grant. Next we will detail the most significant results obtained, and finally we will shortly detail the way how the scientific results of this grant were disseminated at the conferences/research stages financed from this grant. All the details of this grant can be consulted at the **new webpage** of the grant

`http://www.unibuc.ro/~dstamate/grantPCE-2011-3-1023/indexDP.html`

We want to mention that the former webpage of this grant, hosted at the address

`https://dl.dropboxusercontent.com/u/112281424/grantPCE-2011-3-1023/indexDP.html`

had to be changed to the new address after the website dropbox.com changed the rules for editing the webpages. However, both webpages were constantly updated to the latest information about the grant.

I. Scientific Activity. Within this project there were written 45 scientific papers along the lines of the main and secondary objectives. Out of these 45 papers 30 are published, 4 accepted for publication, and 11 (out of which 8 are written in 2016) are submitted for publication. From the 34 papers published/accepted for publication 27 are in ISI journals, 5 are in non-ISI journals and 2 are in Springer volumes of some prestigious conferences. The 27 papers are published or accepted for publication in journals such as **Proceedings of the American Mathematical Society, Manuscripta Mathematica, Michigan Mathematical Journal, Journal of Algebraic Combinatorics, Kyoto Journal of Mathematics, Journal of Symbolic Computation, Mathematische Nachrichten, Journal of Algebra, Journal of Pure and Applied Algebra, The Electronic Journal of Combinatorics, Experimental Mathematics, Journal of Commutative Algebra, Communications in Algebra, Journal of Algebra and its Applications, Archiv der Mathematik.** We strongly believe that the research activity sustained by this grant is at high parameters in terms of number of publications, but mainly from the quality point of view. First of all, we list all scientific papers written in the frame of this project, and afterward we detail, selectively, the most significant results obtained.

Published papers (the first 26 are in ISI journals):

(1) A. Aslam, V. Ene, *Simplicial complexes with rigid depth*, **Arch. Math.** 99 (4) (2012), 315-325.
(2) M. Cimpoeas, *The Stanley conjecture on monomial almost complete intersection ideals*, **Bull. Math. Soc. Sci. Math. Roumanie** vol 55(103), no. 1 (2012), 35-39.

(3) V. Ene, J. Herzog, T. Hibi, F. Mohammadi, *Determinantal facet ideals*, **Michigan Math. J.** 62, 39–57, 2013.
(4) V. Ene, R. Okazaki, *On the radical of muligraded modules*, **J. Algebra** 388, 10–21, 2013.
(5) V. Ene, A. Qureshi, *Ideals generated by diagonal 2-minors*, **Communications in Algebra** 41(8), 3058–3066, 2013.
(6) V. Ene, A. A. Qureshi, A. Rauf, *Regularity of join-meet ideals of distributive lattices*, **Electron. J. Combinatorics** 20(3) (2013) #P20.
(7) D. Popescu, *Upper bounds of depth of monomial ideals*, **J. Commutative Algebra** 5(2), 323-327, 2013.
(8) D. Popescu, A. Zarojanu, *Depth of some square free monomial ideals*, **Bull. Math. Soc. Sci. Math. Roumanie**, vol 56(104) No. 1, 117-124, 2013.
(9) D. Popescu, A. Zarojanu, *Depth of some special monomial ideals*, **Bull. Math. Soc. Sci. Math. Roumanie**, vol 56 (104) No. 3, 365-368, 2013.
(10) D. Popescu, *Depth of factors of squarefree monomial ideals,* **Proceedings AMS** 142, 1965–1972, 2014.
(11) H. Charalambous, A. Thoma, M. Vladoiu, *Markov complexity of monomial curves,* **J. Algebra** 417, 391–411, 2014.
(12) J. Herzog, M. Vladoiu, *Monomial ideals with primary components given by powers of monomial prime ideals,* **Electronic J. Combinatorics** 21(1), #P1.69, 2014.
(13) B. Ichim, A. Zarojanu, *An algorithm for computing the multigraded Hilbert depth of a module,* **Experimental Mathematics** 23(3), 322–331, 2014.
(14) M. Cimpoeaş, *Stanley depth of quotient of monomial complete intersection ideals,* **Comm. in Algebra** 42, 4274–4280, 2014.
(15) V. Ene, J. Herzog, S. Saeedi Madani, *A note on the regularity of Hibi rings*, **Manuscripta Math.** 148(3), 501-506, 2015.
(16) V. Ene, J. Herzog, T. Hibi, *Linearly related polyominoes*, **J. Algebraic Combinatorics** 41 (4), 949-968, 2015.
(17) V. Ene, J. Herzog, T. Hibi, S. Saeedi Madani, *Pseudo-Gorenstein and level Hibi rings*, **J. Algebra** 431, 138-161, 2015.
(18) V. Ene, J. Herzog, T. Hibi, *Linear flags and Koszul filtrations*, **Kyoto J. Math.** 55(3), 517-530, 2015.
(19) F. Chaudhry, A. Dokuyucu, V. Ene, *Binomial edge ideals and rational normal scrolls*, **Bull. Iranian Math. Soc.** 41, no. 4, 971-979, 2015.
(20) V. Ene, A. Zarojanu, *On the regularity of binomial edge ideals*, **Math. Nachrichten** 288(1), 19-24, 2015.
(21) D. Popescu, A. Zarojanu, *Three generated, squarefree, monomial ideals*, **Bull. Math. Soc. Sci. Math. Roumanie**, 58(106), no 3, 359-368, 2015.
(22) D. Popescu, *Stanley depth on five generated, squarefree, monomial ideals*, **Bull. Math. Soc. Sci. Math. Roumanie**, 59(107), no 1, 75–99, 2016.
(23) A. Dimca, D. Popescu, *Hilbert series and Lefschetz properties of dimension one almost complete intersections*, **Communications in Algebra** 44, 4467–4482, 2016.
(24) D. Popescu, *Depth in a pathological case*, **Bull. Math. Soc. Sci. Math. Roumanie** vol. 59(107), no.2, 187–195, 2016.

(25) M. Cimpoeas, D. Stamate, *On intersections of complete intersection ideals*, **Journal of Pure and Applied Algebra**, vol 220, no. 11, 3702–3712, 2016.
(26) G. Pfister, D. Popescu, *Constructive General Neron Desingularization for one dimensional local rings*, **J. Symbolic Computation** vol. 80, 570–580, 2017.
(27) M. Cimpoeas, *Several inequalities regarding Stanley depth*, Romanian Journal of Mathematics and Computer Science , vol 2, no. 1, 28–40, 2012.
(28) M. Cimpoeas, *Vertex cover algebras of simplicial multicomplexes*, Romanian Journal of Mathematics and Computer Science , vol 3, no. 1, 1–4, 2013.
(29) V. Ene, *Syzygies of Hibi rings*, **Acta Mathematica Vietnamica**, Special Issue on: Commutative Algebra and its Interaction with Algebraic Geometry and Combinatorics II 40(3), 403-446, 2015.
(30) V. Ene, J. Herzog, T. Hibi, *Koszul binomial edge ideals* Bridging Algebra, Geometry, and Topology, **Springer Proceedings in Mathematics & Statistics**, 96, D. Ibadula, W. Veys (Eds.) Springer, 127–138, 2014.

Papers accepted for publication:

(1) D. Popescu, *Around General Neron Desingularization*, to appear in **Journal of Algebra and Its Applications**, 16, No. 22017). Preprint version available at: arXiv:1504.06938.
(2) A. Popescu, D. Popescu, *A method to compute the General Neron Desingularization in the frame of one dimensional local domains*, to appear in Singularities and Computer Algebra-Festschrift for Gert-Martin Greuel on the Occasion of his 70th Birthday, Editors Wolfram Decker, Gerhard Pfister, Mathias Schulze, Springer Monograph. Preprint version available at: arXiv:1508.05511.
(3) M. Cimpoeas, *On the Stanley depth of the path ideal of a cycle graph*, 5 pp, to appear in Romanian Journal of Mathematics and Computer Sciences. Preprint version available at arXiv:1601.00261.
(4) D. Popescu, *Artin approximation property and the General Neron Desingularization*, to appear in Revue Roum. Math. Pures et Appl. Preprint version available at arXiv:1511.06967.

Papers submitted for publication:

(1) D. Popescu, *Nested Artin Approximation*, arXiv:1601.06654.
(2) F. J. Castro–Jimenez, D. Popescu, G. Rond, *Linear nested Artin approximation for algebraic power series*, arXiv:1511.09275.
(3) D. Popescu, *Size and Stanley depth of monomial ideals*, arXiv:1602.06760.
(4) M. Cimpoeas, *On the Stanley depth of a special class of Borel type ideals*, 7pp, arXiv:1603.03939.
(5) M. Cimpoeas, *A class of square-free monomial ideals associated to two integer sequences*, arXiv.1604.02933.
(6) M. Cimpoeas, F. Nicolae, *On the restricted partition function*, arXiv.1609.06090.
(7) M. Cimpoeas, F. Nicolae, *On the restricted partition function, II*, arXiv.1611.00256.
(8) M. Cimpoeas, D. Stamate, *Groebner-nice pairs of ideals*, in preparation.
(9) M. Cimpoeas, *Stanley depth of the path ideal associated to a line graph*, arXiv:1508.07540.
(10) M. Cimpoeas, *On the quasi-depth of squarefree monomial ideals and the sdepth of the monomial ideal of independent sets of a graph*, arXiv:1511.06974.

(11) M. Cimpoeas, *On the Stanley depth of powers of some classes of monomial ideals*, arXiv:1512.08195.

II. The most important scientific results: In the following we sketch the most significant results obtained in the papers already published within this grant.

In the paper **(1)** from the published papers the authors characterize the unmixed monomial ideals $I \subset S = K[x_1,\ldots,x_n]$ with $\operatorname{depth}(S/I) = \operatorname{depth}(S/\operatorname{rad}(I))$, that is, those ideals of maximal depth. The depth of S/I is maximal since it is known that the Betti numbers decrese by passing to the radical of I, therefore the depth of S/I increases when we pass to $\sqrt{I}$. In addition, as an application of the above result, a class of simplicial complexes called "with rigid depth" is characterized. We say that a pure simplicial complex has *rigid depth* if for every unmixed monomial ideal $I \subset S$ with $\sqrt{I} = I_\Delta$ one has $\operatorname{depth}(S/I) = \operatorname{depth}(S/I_\Delta)$. The rigid depth simplicial complexes generalize in a natural way the simplicial complexes studied by J. Herzog, Y. Takayama, N. Terai. In particular, from this characterization, it follows that if a pure simplicial complex has rigid depth over a field of characteristic 0, then it has rigid depth over any field.

In the paper **(3)** the authors consider ideals generated by general sets of m-minors of an $m \times n$-matrix of indeterminates. The generators are identified with the facets of an $(m-1)$-dimensional pure simplicial complex. The ideal generated by the minors corresponding to the facets of such a complex is called a determinantal facet ideal. When $m = 2$, which means that Δ is a graph, the ideal J_Δ is a binomial edge ideal. These binomial ideals were introduced in 2010 and were intensively studied in the recent years. The determinantal facet ideals have a much more complicated structure than the binomial edge ideals. In this paper, it is discussed the question when the generating minors of its determinantal facet ideal J_Δ form a Gröbner basis and when J_Δ is a prime ideal. It is shown that the generators form a Gröbner basis with respect to the lexicographic order induced by the natural order of the variables if and only if Δ is closed, a combinatorial property which generalizes somehow the closed graphs. When Δ is closed, it is shown that J_Δ is Cohen-Macaulay and the K–algebra generated by the generators of J_Δ is Gorenstein. For Δ closed, a necessary condition for the primality of J_Δ is given. This condition is expressed in terms of combinatorics of Δ. Under additional conditions on Δ sufficient conditions for the primality of J_Δ are given.

In the paper **(4)** it is defined a functor $\mathfrak{r}^*$ from the category of positive determined modules to the category of the squarefree modules which plays a similar role to taking the radical for monomial ideals. It was already known that the Betti numbers do not increase when one passes from a monomial ideal to its radical. The authors show that passing from a positively $\mathbf{t}$-determined module to its "radical" module there is a similar behavior. In particular, one obtains $\operatorname{depth} M \leq \operatorname{depth} \mathfrak{r}^* M$ for any positively $\mathbf{t}$-determined module M. Unlike the monomial case, for a positively $\mathbf{t}$-determined module M, we show that one has only the inequality $\dim \mathfrak{r}^* M \leq \dim M$. Easy examples show that the inequality may be strict. By using the inequalities between depth and Krull dimension, it is shown that the (sequentially) Cohen-Macaulay property of M passes to the "radical" of M for any positively $\mathbf{t}$-determined module M with $\mathfrak{r}^* M \neq 0$. Moreover, there are studied the connections between the functor $\mathfrak{r}^*$ and the functors Ext, the Alexander dual (first introduced and studied by E.Miller in 2000) and the Auslander-Reiten translate functor introduced by Brun and Floystad în 2011. The connection between r^* and Ext allows to prove that if M and $\mathfrak{r}^*(M)$ have the same

Krull dimension, then M is generalized Cohen-Macaulay if and only if $r^*(M)$ is so and if M is Buchsbaum, then the radical of M is also Buchsbaum.

The papers **(10), (7) and (8)** present particular cases when Stanley's conjecture for the squarefree monomial ideals holds. In the paper (10), it is considered the case of a monomial ideal generated by r squarefree monomials of degree d. The author proves that if r is greater or equal than the number of squarefree monomials of I of degree $d+1$, then $\operatorname{depth} S/I = d$. If J is a nonzero monomial ideal properly contained in I, generated by squarefree monomials of degree greater or equal than $d+1$, and r is strictly bigger than the number of squarefree monomials of I/J of degree $d+1$ (or more generally $\operatorname{sdepth} I/J = d$) atunci $\operatorname{depth} I/J = d$. In particular, the author obtains in the situations described above a positive answer for Stanley's conjecture. The main result (Theorem 2.2) gives a sufficient condition, namely $\rho_d(I) > \rho_{d+1}(I) - \rho_{d+1}(J)$, that implies $\operatorname{depth} I/J = d$. Here, $\rho_d(I)$ represents the number of all squarefree monomials of degree d of I. The proof of this result makes use of Koszul homology, a new technique to tackle this conjecture, introduced by the author. Moreover, the author explains why this technique seems to be better suited for this particular cases of the conjecture. If I is generated by at least $\rho_{d+1}(I)$ squarefree monomials of degree d, then it is proved in Corollary 3.4 that $\operatorname{depth} I = d$. This generalizes a previous result of the same author, the starting point for this research paper. In addition, it is also shown that the imposed conditions are consequences of the fact that $\operatorname{sdepth} I/J = d$, which means that Stanley's conjecture holds in this case. În lucrarea (7), autorul reuşeşte calculul unor margini superioare pentru , care satisfac anumite ipoteze suplimentare. In the paper (7), the author computes some upper bounds for $\operatorname{depth} I/J$ when $J \subset I$ are squarefree monomial ideals satisfying some extra hypothesis. In the paper (8), the authors prove new cases when Stanley's conjecture holds true. More precisely, they consider the case when I and J are two squarefree monomial ideals such that J is properly contained in I, the minimal generators of I are of degree ≥ 1, and J in degree ≥ 2. In addition, if I contains exactly one variable among the minimal generators, and the other generators are of degree greater than or equal to 2 then $\operatorname{sdepth} I/J \leq 2$ implies that $\operatorname{depth} I/J \leq 2$, thus in particular Stanley's conjecture holds true. In order to prove the main result, Theorem 1.10, the authors extend the previous results and techniques from (10) and (7).

In the paper **(20)** it is shown that if G is a closed connected graph (that staisfies a certain combinatorial condition which, from algebraic point of view is equivalent to the fact that J_G has a quadratic Gröbner basis) the regularity of S/J_G is equal to the length of the longest induced path in G. Therefore, Matsuda-Murai conjecture is true for closed graphs. In the same paper, it is shown that this conjecture is true for aclass of graphs which includes the trees. In addition, it is shown that for G closed, the regularity of J_G coincides with the regularity of the initial ideal of J_G with respect to the lexicographic order. This result supports a recent conjecture, formulated by V. Ene, J. Herzog and T. Hibi, which claims that J_G and its initial ideal with respect to lex order have the same extremal Betti numbers.

In the paper **(6)**, the authors consider binomial ideals associated with distributive lattices. Given a distributive lattice L in the polynomial ring $K[L]$ they consider the binomial ideal I_L called join-meet ideal which is generated by the binomials of the form $ab-(a\vee b)(a\wedge b)$, where a and b are incomparable elements in L. These ideals are also called Hibi ideals since they are the defining ideals of the Hibi rings. These rings were introduced and studied by Hibi in a series of papers from the end of 80's. The paper (6) approaches for the first time in literature the study of syzygies of Hibi ideals. For a distributive planar lattice L, it is shown that the regularity of the associated Hibi ideal can be expressed in terms of the combinatorics of the lattice. For an arbitrary non-planar lattice L, bounds for the regularity of I_L are obtained. More precisely, it is shown that the regularity of I_L is greater than or equal to the number of join-irreducible elements of L which are pairwise incomparable minus 1 and smaller than or equal to the number of join-irreducible elements of L minus 1. As an application, it is proved that I_L has a linear resolution if and only if the lattice is isomorphic to the divisor lattice of $2 \cdot 3^a$, with $a \geq 1$.

In the paper **(13)**, the authors introduce an algorithm for computing the Hilbert depth of a finitely generated multigraded module M over the standard multigraded polynomial ring $R = K[X_1, \ldots, X_n]$. The algorithm is based on the method introduced by B. Ichim and J.J. Moyano-Fernandez with several improvements. It may also be adapted for computing the Stanley depth of M if $\dim_K M_a \leq 1$ for all $a \in \mathbb{Z}^n$. Further, they provide an experimental implementation of the algorithm in CoCoA and they use it to find interesting examples, which in some cases offer complete answers to some problems posed by J. Herzog in 2013.

In the paper **(11)** are studied the toric ideals of monomial curves in $\mathbb{A}^3$. These toric ideals $I_{\mathscr{A}} \subset K[x_1, x_2, x_3]$ were studied for the first time by Herzog in 1970, who proved that $I_{\mathscr{A}}$ is either complete intersection (in which case is minimally generated by 2 binomials) or almost complete intersection (in which case is minimally generated by 3 binomials). Toric ideals are binomial ideals, whose connection with algebraic statistics was for the first time studied in the seminal paper of Diaconis and Sturmfels from 1996, generating ever since an ongoing research activity in this field. Of a particular importance in algebraic statistics are the following invariants: Markov complexity $m(\mathscr{A})$ and Graver complexity $g(\mathscr{A})$ in the case when $\mathscr{A}$ is a finite set of vectors $\mathbf{a}_1, \ldots, \mathbf{a}_r$ from $\mathbb{N}^n$, where $r \geq 3$ and $n \geq 1$. The fundamental result in this direction is that $g(\mathscr{A}) < \infty$ and a formula of computing it, given by F. Santos si B. Sturmfels in 2004. For the Markov complexity, which is much more important from the point of view of applications in algebraic statistics, it is only known that is bounded above by the Graver complexity. However, a formula for computing it is still unknown. Moreover, the Markov complexity is known only in a few particular cases. Santos and Sturmfels leave in 2004 as an open question the computation of the Markov complexity $m(\mathscr{A})$ in the case when $\mathscr{A} = \{n_1, n_2, n_3\}$ in terms of n_1, n_2, n_3, and conjecture that $g(\mathscr{A}) = n_1 + n_2 + n_3$ if $\gcd(n_1, n_2) = \gcd(n_1, n_3) = \gcd(n_2, n_3) = 1$. Unexpectedly, the authors prove in (11) that $m(\mathscr{A}) = 3$ if $I_{\mathscr{A}}$ is almost complete intersection, while $m(\mathscr{A}) = 2$ if $I_{\mathscr{A}}$ is complete intersection. In addition, the authors prove that the conjectured value for $g(\mathscr{A})$ turns out to be wrong. However, the following inequality $g(\mathscr{A}) \geq n_1 + n_2 + n_3$ holds in general. The more surprising the results are since the Graver complexity may be as large as possible, while the Markov complexity is at most 3.

In the paper **(12)** the authors study the monomial ideals which can be written as intersection of powers of monomial prime ideals, which they call *monomial ideals of intersection type*. It is a known fact that every squarefree monomial ideal is of intersection type, being the irredundant intersection of its minimal monomial prime ideals. Obviously, among the non-radical monomial ideals, the closest to the squarefree monomial ideals are the monomial ideals of intersection type. The authors succeed to characterize in Theorem 1.1 all the monomial ideals $I \subset S$ which are of intersection type. More precisely, I is of intersection type if and only if for every associated prime $\mathfrak{p}$ of I the minimal degree of a generator of the monomial localization $I(\mathfrak{p})$ of I is greater than or equal to the maximal degree of a nonzero socle element of $S(\mathfrak{p})/I(\mathfrak{p})$. In addition, the authors also prove that if I is of intersection type, its presentation as intersection of powers of monomial prime ideals is unique. The exponents of the powers of the monomial prime ideals associated to I, in the case of intersection type ideals are bounded above by $\mathrm{reg}(I(\mathfrak{p}))$, for any associated prime $\mathfrak{p}$ of I, see Theorem 1.3. In the case when the exponents are equal to the upper bound for every associated prime $\mathfrak{p}$, then the monomial ideal is called of *strong intersection type* and it is proved that these ideals are exactly those monomial ideals I, for which their monomial localizations $I(\mathfrak{p})$ have linear resolution for each $\mathfrak{p} \in \mathrm{Ass}(S/I)$. One section is dedicated to the general properties of monomial ideals of intersection type. It is proved that they are integrally closed, and the support hyperplanes of the Newton polyhedron of such an ideal can be described in terms of the unique irredundant primary decomposition described above. The class of such ideals contains (as proved by the authors) the polymatroidal ideals and the principal Borel ideals (which are the only Borel type ideals with such property). Another important result is the classification of all edge ideals whose second power is of intersection type. An important consequence of this result is the fact that the graphs with the property that the second power of its edge ideal is not of intersection type, have none of the powers of its edge ideal of intersection type.

In the paper **(18)** the authors study the connection between the property of a standard graded K-algebra R of having Koszul filtrations and the property of its defining ideal I of having quadratic Gröner bases. Let K be a field and $S = K[x_1,\ldots,x_n]$ the polynomial ring in n variables. Let $I \subset (x_1,\ldots,x_n)^2$ be a graded ideal of S. Consider the K-algebra $R = S/I$ and denote by $\mathfrak{m}$ its maximal graded ideal. The algebra R is called Koszul if the residue field $K = R/\mathfrak{m}$ has an R-liniar resolution. It is known that if R is Koszul then I is generated by quadrics, while if I has a quadratic Gröbner basis then R is Koszul. In this article it is studied the connection between the following properties:

(i) R has a Koszul filtration;
(ii) I has a quadratic Gröbner basis.

In Theorem 1.1 it is shown that if I has a quadratic Gröbner basis with respect to the revlex order then R admits a linear flag.

In the paper **(16)** it is continued the study of the polyomino ideals introduced by Qureshi in 2012. This class includes the two-sided ladder determinantal ideals and the Hibi binomial ideals associated to planar distributive lattices. A polyomino ideal is generated by a collection of 2-minors of a generic matrix X of type $m \times n$. In this paper the authors considered such collections of minors associated to a convex configuration. It is a difficult question to understand the resolution of such ideals. The regularity of Hibi rings associated to distributive lattices is computed in (6). Sharpe had proved that the determinantal ideal $I_2(X)$ generated by the 2-minors of the generic matrix X has linear relations. Kurano extended this result to ideals of the form $I_t(X)$ where $2 \leq t \leq \min(m,n)$. Hashimoto proved that, in general, the resolution of such ideals depends on the characteristic of the basefield. However, Bruns and Herzog proved that the second Betti number $\beta_2(I_2(X))$ does not depend on $\mathrm{char}(K)$. Using the square-free divisor complex technique introduced by Bruns and Herzog the authors classify the polyomino ideals with linear relations. Moreover, they determine necessary conditions for the polynomino ideal $I_{\mathscr{P}}$ associated to a polyomino $\mathscr{P}$ to have a linear resolution. In particular, they derive new results concerning the resolution of the Hibi ideals associated to distributive lattices.

In the paper **(15)** the authors prove that for any distributive lattice L one has

$$\operatorname{reg} R[L] = |P| - \operatorname{rank} P - 1.$$

About the Hibi ring $R[L]$ associated to a distributive lattice $L = \mathscr{I}(P)$, where P is a finite poset, it was already known from the work of Hibi that $\dim R[L] = |P| + 1$, hence $\operatorname{proj}\dim R[L] = |L| - |P| - 1$ since $R[L]$ is Cohen-Macaulay. A first attempt to study the regularity of $R[L]$ was made in the paper (6). It is proven there that when P is a planar lattice, the regularity of $R[L]$ equals the maximum number of squares in a cyclic sublattice of L. For the proof of the displayed equality the authors use the combinatorial description given by Hibi for the generators of the canonical ideal of $R[L]$. The formula for $\operatorname{reg} R[L]$ allows the authors to characterize the lattices L for which the regularity of $R[L]$ is 1 or 2.

In the paper **(17)** it is continued the study of Hibi rings and ideals. For a Cohen-Macaulay standard graded algebra R with canonical module ω_R, one knows that R is Gorenstein if and only if ω_R is cyclic. This condition on ω_R may be relaxed in at least two directions. When ω_R is generated in a single degre, R is called level. When there is a single generator for ω_R lying in the smallest degree, we call R pseudo-Gorenstein. The latter concept is introduced here for the first time. In this paper they characterize in terms of the poset P of join-irreducible elements the distributive lattices $L = \mathscr{I}(P)$ such that the corresponding Hibi ring is pseudo-Gorenstein. Moreover are explained the connections of the pseudo-Gorenstein algebras with the level or Gorenstein algebras. They also introduce the concept of hyper-planar lattice, which extends naturally the notion of planar lattice. In the paper they prove that a hyper-planar lattice is pseudo-Gorenstein (i.e. its Hibi ring has this property) if and only if all the chain in a canonical decomposition of P have the same length. A sufficient condition for $R[L]$ to be level was given by Ezra Miller in 2000. This is not usually necessary. In this paper the authors present a necessary condition for $R[L]$ to be level and they conjecture that this is also sufficient. This condition is expressed in terms of the combinatorics of the poset P. For planar lattices satisfying a certain regularity condition they prove that the above mentioned necessary condition is also enough to guarantee the level property.

In the paper **(23)** it is described the Hilbert series of $S/(f)$ în terms of the Hilbert series of S/I, where I is the saturation of (f) in S. We recall that $S = K[x_0, \dots, x_n]$ is a polynomial ring over a field K of characteristic zero and $f = f_0, \dots, f_n$ an almost complete intersection system of polynomials with deg $f_i = d_i$. When I is a complete intersection, one can fully determine the Hilbert series of $S/(f)$. This problem is motivated by singularity theory. Let $V \subset \mathbf{P}^n$ be a projective hypersurface with isolated singularities given by an equation $g = 0$, $g \in S$. Then the partial derivatives $g_0, \dots, g_n$ satisfy the above conditions, hence one can describe the Hilbert series of the Milnor algebra $M(g) = S/(g_0, \dots, g_n)$ associated to g. This may be used to compute various invariants of V. In this context one studies the Lefschetz type properties of the algebra $M(g)$. The

Lefshetz properties have been studied for quite a while. Nevertheless, several conjectures are still open, even in the case when $f_0,\dots,f_n$ is a regular sequence. What is known works for $n \leq 2$. Other several small results are proved for $n > 2$ and this topic seems to be generously funded by the NSA. It is believed that they may be useful in criptography. In the current paper the authors describe a Lefschetz type property when f is an almost complete intersection and $n = 2$, in particular for $M(g)$ in the case when $n = 2$. Counterexamples are available when $n > 2$.

In the paper **(26)** the authors give an algorithmic proof for the General Neron desingularization in the case of the 1-dimensional local rings. In addition, this algorithmic proof can be implemented in Singular, and the authors describe this implementation. They also prove a version of Greenberg's theorem regarding strong approximation property in the case of 1-dimensional local rings.

III. Dissemination of results:

One of the main ways of dissemination of the scientific results by the members of this research team is their presentation at the weekly Commutative Algebra Seminar "Nicolae Radu" run jointly by IMAR and the University of Bucharest hosted every Tuesday during 12-14 by the room 120 of the Faculty of Mathematics and Computer Science. For a precise evidence of all the talks delivered in this seminar by the members of this research team in the period 2012–2016 one can see the archive of these talks on the following webpage:

`http://www.imar.ro/organization/activities/archive/seminars_arh_sem_19_s.php`

Next we briefly present, yearly, the way how the scientific results obtained within this project were disseminated, excepting the presentations from the weakly Commutative Algebra Seminar "Nicolae Radu". The complete list of the delivered talks can be seen on the webpage of this grant.

2012 Dorin Popescu, Viviana Ene, Bogdan Ichim, Dumitru Stamate and Mircea Cimpoeas were the organizers of the 20-th edition of the traditional National School on Algebra, "Discrete invariants in commutative algebra and in algebraic geometry", organized in Mangalia in the period 02.09.2012-08.09.2012, see

`http://math.univ-ovidius.ro/sna/edition.aspx?itemID=6`.

This edition benefited of an impressive international presence (11 invited speakers from abroad) and a strong scientifical impact. Also, Viviana Ene, Bogdan Ichim and Dumitru Stamate have participated with a contributed talk to this school. In addition, Bogdan Ichim gave the talk "Introduction to Normaliz" at Rostock University on 09-05-2012, and another one entitled "How to compute the multigraded Hilbert depth of a module" at Osnabruck University on 20-11-2012. Also Dorin Popescu gave the talk "Contributions and new results on Stanley's conjecture" at Kaiserslautern University in July 2013. Another member of the grant, Viviana Ene, had a reserach stage abroad at Essen University, in the framework of a scientific cooperation with Prof. Jurgen Herzog. Andrei Zarojanu participated to the conference "Workshop for young researchers in Mathematics", which took place in the period 10.05.2012-11.05.2012 at Constanta, with the talk "Stanley conjecture on intersection of three monomial primary ideals".

2013 This year the members of our research team have participated with contributed talks to the following national/international conferences:

1. *Experimental and Theoretical Methods in Algebra, Geometry and Topology*, international conference, Eforie Nord, 21-24 June 2013. The conference benefited of participants from 14 countries, see
   `http://math.univ-ovidius.ro/Conference/ETMAGT60/`
   Dorin Popescu gave at this conference the talk *A hope for Stanley Conjecture on monomial ideals*.
2. *Joint International Meeting of the American Mathematical Society and the Romanian Mathematical Society*, organized at University ”1 Decembrie 1918” from Alba Iulia, 27-30 June 2013.
   `http://imar.ro/ams-ro2013/description.php`
   Viviana Ene gave at this conference the talk *Binomial ideals and graphs* .
3. The anniversary conference *Faculty of Sciences - 150 years* which took place in the period 29.08-01.09.2013, at the University of Bucharest.
   `http://fmi.unibuc.ro/FMI-150/`
   Dorin Popescu gave the talk *Around Stanley's conjecture on monomial ideals*, while Viviana Ene spoke about *Binomial edge ideals*.

4 The 21-st edition of the National School on Algebra, ”Algebraic Methods in Combinatorics”, organized at IMAR, in the period 2-6 September 2013, see
   `http://math.univ-ovidius.ro/sna/edition.aspx?itemID=7`.
   At this school participated a big number of master students and Ph. D. students from Romania, as well as from abroad.
   Some members of the grant gave the following talks (some of them accompanied also by tutorials):
   - *Algebraic and homological properties of binomial edge ideals* (2 lectures), Viviana Ene,
   - *Tools of Combinatorial Commutative Algebra*, Dumitru Stamate
   - *Matroids and realisability*, Dumitru Stamate
   - *Polymatroidal ideals* (2 lectures), Marius Vladoiu

   There were also presented the following short contribution talks:
   - *Stanley depth of quotient of monomial complete intersection ideals*, Mircea Cimpoeas
   - *Depth of some special monomial ideals*, Andrei Zarojanu.

2014 The members of the research team presented the following talks at the 2014 edition of the National School on Algebra that took part at IMAR, during September 1-5.
(see `http://math.univ-ovidius.ro/sna/edition.aspx?cat=GeneralInfo&itemID=8`):

(1) Viviana Ene, ”Hibi rings and their Grobner bases”, 1.09.2014.
(2) Miruna Roşca, ”Vertex cover algebras of weighted graphs”, 1.09.2014.
(3) Viviana Ene, ”Level and pseudo - Gorenstein Hibi rings”, 2.09.2014.
(4) Andrei Zarojanu, ”An algorithm for computing the multigraded Hilbert depth of a module”, 2.09.2014.
(5) Viviana Ene, ”The regularity of Hibi rings”, 4.09.2014.

We would also like to mention that some of the team members were among the organizers of this edition:Viviana Ene, Miruna Roşca, Andrei Zarojanu, Dumitru Stamate and Marius Vlădoiu.

2015 The members of the research team had several international/national talks during 2015. Undoubtedly, the most prestigious international exposure this year belongs to the director of this project who was invited speaker for the special semester dedicated to Artin Approximation and Singularity Theory at CIRM, Luminy (France)

(see `http://chairejeanmorlet-1stsemester2015.weebly.com/program-overview.html`)

He was invited to deliver a four lecture series as part of a mini-course for Ph.D students in the period 26-30 January 2015, with the theme "Introduction to Artin Approximation and the Geometry of Power Series Spaces". The titles of the four lectures are: 1) Smooth morphisms, 2) The smooth locus of a morphism, 3) Elkik's theorem, and 4) General Neron Desingularization in dimension 1. Later on, he was invited speaker at the conference "Applications of Artin Approximation in Singularity Theory", held in the period February 2-6, 2015, giving a talk on "Artin approximation, versal deformations, and maximal Cohen-Macaulay modules". Another talk presented by Dorin Popescu was "On the regular local rings" at the conference "Artin Approximation and Infinite Dimensional Geometry" organized in the period 23-27 March 2015 at CIRM, Luminy. His presentations were highly appreciated and consequently he was invited in the period 2.11-10.11.2015 for a research stage at the prestigious university KU Leuven, where he also delivered a talk in the general seminar on the 4th of November.

Every member of our research team has delivered a talk at "The Eighth Congress of Romanian Mathematicians" held in Iaşi, Romania, June 26 - July 1, as a proof of the international research visibility of our team,

(see `http://www.math.uaic.ro/cmr2015/programme/Algebra%20and%20Number%20Theory.pdf`)

In addition, the members of the research team support scientifically, as part of the tradition during the last years, the National School on Algebra. This year, the 23rd edition of the National School on Algebra took place at IMAR, during the period 31.08-04.09. The lectures delivered by the members of our team can be seen at the following

(see `http://math.univ-ovidius.ro/sna/edition.aspx?cat=GeneralInfo&itemID=9`)

We would also like to mention that three of the team members were among the organizers of this edition: Viviana Ene, Dumitru Stamate and Marius Vlădoiu.

2016 This year Viviana Ene, Dumitru Stamate and Marius Vlădoiu organized the 24-th edition of the National School on Algebra "EMS Summer School on Multigraded Algebra and Applications with support from Foundation Compositio Mathematica" at Moieciu de Sus, in the perioad 17 Aug - 24 Aug 2016. The scientific committee of this conference consisted of Viviana Ene, Juergen Herzog (Univ. Duisburg-Essen University, Germania), Thomas Kahle (Magdeburg Univ., Germania), Ezra Miller (Duke University, USA), Apostolos Thoma (Ioannina Univ., Grecia).

Along this year the members of our research team have also delivered the following talks:

- Mircea Cimpoeas, Stanley depth of a class of square-free monomial ideals, at ”Workshop for Young Researchers in Mathematics”, Constanţa, România, 19-22 May 2016.
- Dumitru Stamate, Periodic properties in the shifted family of a numerical semigroup, at ”Workshop for Young Researchers in Mathematics” Constanţa, România, 19-22 May 2016.
- Mircea Cimpoeas, On intersection of complete intersection ideals, at ”International meeting on numerical semigroups with applications”, Levico Terme, Italia, 4-8 July 2016.
- Dumitru Stamate, Quadratic numerical semigroups and the Koszul property, at ”International meeting on numerical semigroups with applications”, Levico Terme, Italia, 4-8 July 2016.
- Dumitru Stamate, Quadratic numerical semigroups and the Koszul property, in the algebra seminar of Univ. Osnabrueck, Germania, 27 September 2016.
- Dumitru Stamate, Betti numbers for numerical semigroup rings, at ”Workshop and Conference on Commutative Algebra”, Tribhuvan University, Kirtipur, Kathmandu, Nepal, 20 Oct 2016.
- Dumitru Stamate, On the Koszul property for numerical semigroup rings, at ”Workshop and Conference on Commutative Algebra”, Tribhuvan University, Kirtipur, Kathmandu, Nepal, 24 Oct 2016.
- Marius Vlădoiu, The defining matrices of self-dual projective toric varieties, in the algebra seminar of Univ. Bilkent, Ankara, Turcia, 1 Dec. 2016,

  (see `http://www.fen.bilkent.edu.tr/~cvmath/sem-now.html`).

Director of the grant,

Prof. dr. Dorin Popescu