On the interactive visualization of implicit surfaces

Geometry meets Computer Graphics

Peter Schenzel

Department of Computer Science, Martin-Luther University Halle

September 6, 2012, Mangalia

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

Ministry of Education and Science: Year of Science

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

Ministry of Education and Science: Year of Science

2008 Year of Mathematics

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

- Ministry of Education and Science: Year of Science
- 2008 Year of Mathematics
- Popularization of Mathematics for young people

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

- Ministry of Education and Science: Year of Science
- 2008 Year of Mathematics
- Popularization of Mathematics for young people
- Mathematical Research Institute (Oberwolfach): Exibition Imaginary

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

- Ministry of Education and Science: Year of Science
- 2008 Year of Mathematics
- Popularization of Mathematics for young people
- Mathematical Research Institute (Oberwolfach): Exibition Imaginary

2008 – Year of Mathematics in Germany

With the eyes of Mathematics

- Ministry of Education and Science: Year of Science
- 2008 Year of Mathematics
- Popularization of Mathematics for young people
- Mathematical Research Institute (Oberwolfach): Exibition Imaginary

IMAGINARY 2008-2010
Eine Wanderausstellung des Mathematischen Forschungsinstituts Oberwolfach

Wissenschaftsjahr 2008
Mathematik
Alles, was zählt

Bericht vom 01.09.2010

Kontakt:
Mathematisches Forschungsinstitut Oberwolfach
Schwarzwaldstr. 9-11
D-77709 Oberwolfach-Walke
Deutschland
Tel: 07834/979-0
Email: greuel@mfo.de
Webseite: www.mfo.de / www.imaginary-exhibition.com

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

Leitung: Gert-Martin Greuel, greuel@mfo.de
Koordination: Andreas Matt, matt@mfo.de
Mitarbeit: Herwig Hauser, Christian Stussak, Oliver Labs, Henning Meyer, Felix Riemann, Steffen Weissmann, Sebstía Xambó
Inhaltliche Gestaltung: Aurelién Alvarez, Luc Benard, Martin von Gagern, Éthienne Ghys, Stephan Klaus, Manfred Kuhnkies, Jos Leys, Richard Palais, Jürgen Richter-Gebert, Ulrich Pinkall, Nicholas Schmitt,
Partner: Matheon / TU Berlin, RSME, TU München, TU Kaiserslautern, Universität Halle, Institut Forwiss/Universität Passau
Förderung/Sponsoren: Bundesministerium für Bildung und Forschung, Alphaform, Voxeljet

"Classical" mathematical institutes

Collections of mathematical models

"Classical" mathematical institutes

Collections of mathematical models

Influence of Felix Klein

"Classical" mathematical institutes

Collections of mathematical models

- Influence of Felix Klein
- Martin Luther Univ. ~ 500 objects

"Classical" mathematical institutes

Collections of mathematical models

- Influence of Felix Klein
- Martin Luther Univ. $\sim$ 500 objects
- Concrete Mathematical Surfaces

"Classical" mathematical institutes

Collections of mathematical models

- Influence of Felix Klein
- Martin Luther Univ. $\sim$ 500 objects
- Concrete Mathematical Surfaces
- Modells made by gypsum

"Classical" mathematical institutes

Collections of mathematical models

- Influence of Felix Klein
- Martin Luther Univ. $\sim$ 500 objects
- Concrete Mathematical Surfaces
- Modells made by gypsum

"Classical" mathematical institutes

Collections of mathematical models

- Influence of Felix Klein
- Martin Luther Univ. $\sim$ 500 objects
- Concrete Mathematical Surfaces
- Modells made by gypsum

Tangent variety of the twisted cubic

Affine twisted cubic

1. Affine equation:
$x = t, y = t^2, z = t^3$

Tangent variety of the twisted cubic

Affine twisted cubic

1. Affine equation:
   $x = t, y = t^2, z = t^3$
2. Tangent variety:
   $x = t + u, y = t^2 + 2tu, z = t^3 + 3t^2u$

Tangent variety of the twisted cubic

Affine twisted cubic

1. Affine equation:
$x = t, y = t^2, z = t^3$
2. Tangent variety:
$x = t + u, y = t^2 + 2tu, z = t^3 + 3t^2u$
3. Implicit form:
$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$

Tangent variety of the twisted cubic

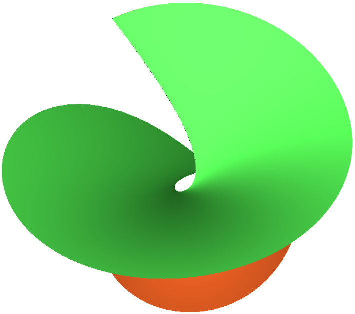
Affine twisted cubic

1. Affine equation:
$x = t, y = t^2, z = t^3$
2. Tangent variety:
$x = t + u, y = t^2 + 2tu, z = t^3 + 3t^2u$
3. Implicit form:
$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$
4. (obtained by elimination)

Tangent variety of the twisted cubic

Affine twisted cubic

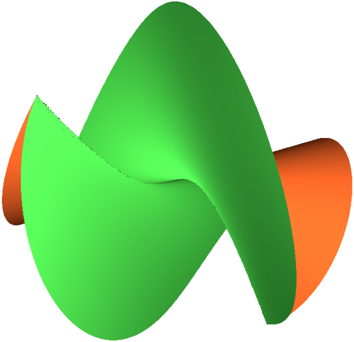
1. Affine equation:
$x = t, y = t^2, z = t^3$
2. Tangent variety:
$x = t + u, y = t^2 + 2tu, z = t^3 + 3t^2 u$
3. Implicit form:
$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$
4. (obtained by elimination)



Tangent variety of the twisted cubic

Affine twisted cubic

1. Affine equation:
   $x = t, y = t^2, z = t^3$
2. Tangent variety:
   $x = t + u, y = t^2 + 2tu, z = t^3 + 3t^2u$
3. Implicit form:
   $F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$
4. (obtained by elimination)



Mathematica

$$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$$

1. Mathematica
   rendering

MATHEMATICA

$$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$$

1. MATHEMATICA rendering
2. implicit plot

MATHEMATICA

$$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$$

1. MATHEMATICA rendering
2. implicit plot
3. polygonal meshes

Mathematica

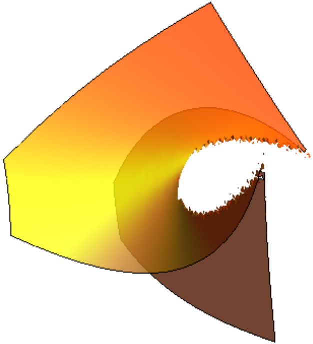
$$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$$

1. Mathematica rendering
2. implicit plot
3. polygonal meshes

Mathematica

$$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$$

1. Mathematica rendering
2. implicit plot
3. polygonal meshes



Mathematica

$$F = 3x^2y^2 - 4x^3z - 4y^3 + 6xyz - z^2$$

1. Mathematica rendering
2. implicit plot
3. polygonal meshes

Examples of visualizations II

- G. Fischer (Hrsg.): *Mathematische Modelle – Aus den Sammlungen von Universitäten und Museen* Bildbd: XII,129 S., 132 Fotos, Kommentarbd: VIII, 89 S., 90 Fig. Geb. im Schuber. Vieweg, 1986.

Examples of visualizations II

- G. Fischer (Hrsg.): *Mathematische Modelle – Aus den Sammlungen von Universitäten und Museen* Bildbd: XII,129 S., 132 Fotos, Kommentarbd: VIII, 89 S., 90 Fig. Geb. im Schuber. Vieweg, 1986.
- S. Endrass a.o.: Surf, `http://surf.sourceforge.net`. Raytracer for algebraic surfaces for static images.

Examples of visualizations II

- G. Fischer (Hrsg.): *Mathematische Modelle – Aus den Sammlungen von Universitäten und Museen* Bildbd: XII,129 S., 132 Fotos, Kommentarbd: VIII, 89 S., 90 Fig. Geb. im Schuber. Vieweg, 1986.
- S. Endrass a.o.: Surf, `http://surf.sourceforge.net`. Raytracer for algebraic surfaces for static images.
- H. Hauser: *Animation von Flächen,* `http://homepage.univie.ac.at/herwig.hauser`, 2005. Animated images made by POV-Ray - a free raytracer.

Examples of visualizations II

- G. Fischer (Hrsg.): *Mathematische Modelle – Aus den Sammlungen von Universitäten und Museen* Bildbd: XII,129 S., 132 Fotos, Kommentarbd: VIII, 89 S., 90 Fig. Geb. im Schuber. Vieweg, 1986.
- S. Endrass a.o.: Surf, `http://surf.sourceforge.net`. Raytracer for algebraic surfaces for static images.
- H. Hauser: *Animation von Flächen,* `http://homepage.univie.ac.at/herwig.hauser`, 2005. Animated images made by POV-Ray - a free raytracer.
- G.-M. Greuel a.o.: Imaginary, `http://www.imaginary2008.de`, 2008. Virtual museum with animated graphics, based on Surfer, creates animated images in the background.

RealSurf I

C. Stussak: *Echtzeit-Raytracing algebraischer Flächen auf der GPU*, Diploma Thesis, University Halle, http://realsurf.informatik.uni-halle.de.

RealSurf I

C. Stussak: *Echtzeit-Raytracing algebraischer Flächen auf der GPU*, Diploma Thesis, University Halle, `http://realsurf.informatik.uni-halle.de`.

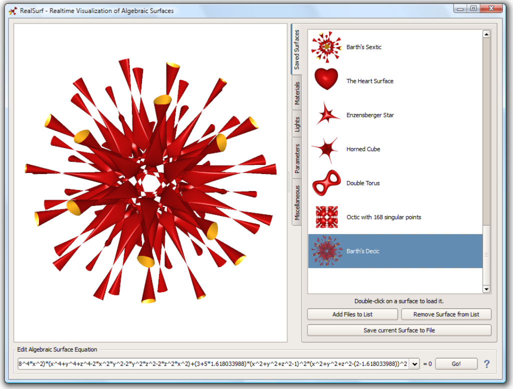


Figure : RealSurf, including a gallery of predefined surfaces

RealSurf II

Remark

Program RealSurf

- *Interactive visualization of algebraic surfaces*

RealSurf II

Remark

Program RealSurf

- *Interactive visualization of algebraic surfaces*
- $F = \sum_{i,j,k} a_{ijk} x^i y^j z^k \in \mathbb{R}[x, y, z]$

RealSurf II

Remark

Program RealSurf

- *Interactive visualization of algebraic surfaces*
- $F = \sum_{i,j,k} a_{ijk} x^i y^j z^k \in \mathbb{R}[x,y,z]$
- OpenGl-*shading language for GPU-programming*

RealSurf II

Remark

Program RealSurf

- *Interactive visualization of algebraic surfaces*
- $F = \sum_{i,j,k} a_{ijk} x^i y^j z^k \in \mathbb{R}[x, y, z]$
- OpenGl*-shading language for GPU-programming*
- *requires* nVidia*-graphic cards series* GeForce *7,8,...*

RealSurf II

Remark

Program RealSurf

- *Interactive visualization of algebraic surfaces*
- $F = \sum_{i,j,k} a_{ijk} x^i y^j z^k \in \mathbb{R}[x, y, z]$
- OpenGl-*shading language for GPU-programming*
- *requires* nVidia-*graphic cards series* GeForce *7,8,...*
- *works proper for surfaces of degree* ≤ 13

Modeling I

1. $G = x^2 + y^2 + z^2 - \beta^2$

Modeling I

1. $G = x^2 + y^2 + z^2 - \beta^2$
2. $F = (\alpha^2 x^2 - y^2)(\alpha^2 y^2 - z^2)(\alpha^2 z^2 - x^2) - (1 + 2\alpha)(x^2 + y^2 + z^2 - 1)^3$, $\alpha = \frac{1}{2}(1 + \sqrt{5})$

Modeling I

1. $G = x^2 + y^2 + z^2 - \beta^2$
2. $F = (\alpha^2 x^2 - y^2)(\alpha^2 y^2 - z^2)(\alpha^2 z^2 - x^2) - (1 + 2\alpha)(x^2 + y^2 + z^2 - 1)^3$, $\alpha = \frac{1}{2}(1 + \sqrt{5})$

Modeling I

1. $G = x^2 + y^2 + z^2 - \beta^2$
2. $F = (\alpha^2 x^2 - y^2)(\alpha^2 y^2 - z^2)(\alpha^2 z^2 - x^2) - (1 + 2\alpha)(x^2 + y^2 + z^2 - 1)^3$, $\alpha = \frac{1}{2}(1 + \sqrt{5})$

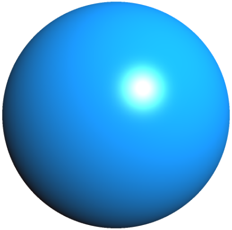


Figure : $G = 0$

Modeling I

1. $G = x^2 + y^2 + z^2 - \beta^2$
2. $F = (\alpha^2 x^2 - y^2)(\alpha^2 y^2 - z^2)(\alpha^2 z^2 - x^2) - (1 + 2\alpha)(x^2 + y^2 + z^2 - 1)^3$, $\alpha = \frac{1}{2}(1 + \sqrt{5})$

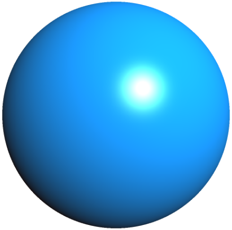


Figure : $G = 0$

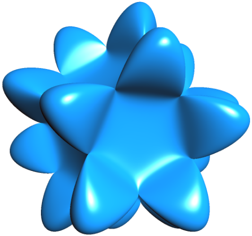


Figure : $F = 0$

Modeling II

Generation of level sets $F \cdot G + \gamma = 0$

Modeling II

Generation of level sets $F \cdot G + \gamma = 0$

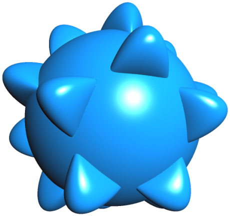


Figure : $\gamma = 0$

Modeling II

Generation of level sets $F \cdot G + \gamma = 0$

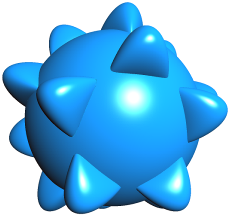


Figure : $\gamma = 0$

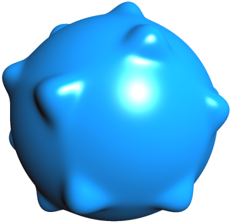


Figure : $\gamma = 0.07$

Modeling II

Generation of level sets $F \cdot G + \gamma = 0$

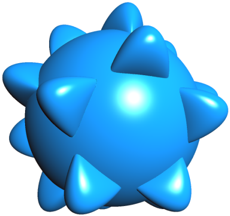


Figure : $\gamma = 0$

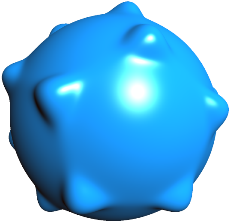


Figure : $\gamma = 0.07$

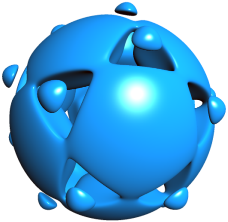


Figure : $\gamma = -0.07$

Modeling III

Intersections with offsets $F^2 + G^2 - \gamma = 0$

Modeling III

Intersections with offsets $F^2 + G^2 - \gamma = 0$

Figure : $\gamma = 0$

Modeling III

Intersections with offsets $F^2 + G^2 - \gamma = 0$

Figure : $\gamma = 0$

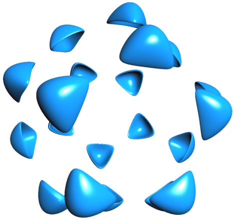


Figure : $\gamma = 0.48348$

Modeling III

Intersections with offsets $F^2 + G^2 - \gamma = 0$

Figure : $\gamma = 0$

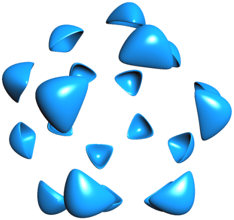


Figure : $\gamma = 0.48348$

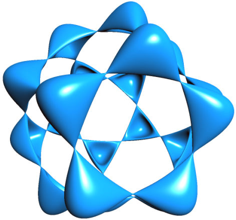


Figure : $\gamma = 0.2$

Deformation of blow ups

Deformation of points: $X = V(f,g) = \{(a,\pm a),(2a,0)\}, f = au^2 + v^2 - 2a^2, g = (u-2a)(u-a)$

Deformation of blow ups

Deformation of points: $X = V(f, g) = \{(a, \pm a), (2a, 0)\}, f = au^2 + v^2 - 2a^2, g = (u - 2a)(u - a)$

Figure : $a = -1/2$

Deformation of blow ups

Deformation of points: $X = V(f,g) = \{(a, \pm a), (2a, 0)\}, f = au^2 + v^2 - 2a^2, g = (u-2a)(u-a)$

Figure : $a = -1/2$

Figure : $a = 0$

Deformation of blow ups

Deformation of points: $X = V(f, g) = \{(a, \pm a), (2a, 0)\}, f = au^2 + v^2 - 2a^2, g = (u - 2a)(u - a)$

Figure : $a = -1/2$

Figure : $a = 0$

Figure : $a = 1/2$

Deformation of equations

Deformation of points: $X = V(f, g) = \{(\pm 1, \pm 1)\}, f = u^2 + ay^2 - a - 1, g = au^2 + y^2 - a - 1, a \neq \pm 1$

Deformation of equations

Deformation of points: $X = V(f,g) = \{(\pm 1, \pm 1)\}, f = u^2 + ay^2 - a - 1, g = au^2 + y^2 - a - 1, a \neq \pm 1$

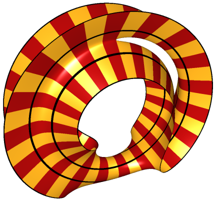


Figure : $a = -1/2$

Deformation of equations

Deformation of points: $X = V(f,g) = \{(\pm 1, \pm 1)\}, f = u^2 + ay^2 - a - 1, g = au^2 + y^2 - a - 1, a \neq \pm 1$

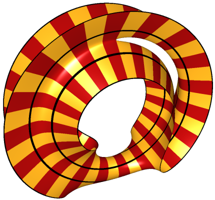


Figure : $a = -1/2$

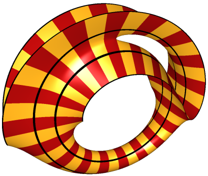


Figure : $a = 0$

Deformation of equations

Deformation of points: $X = V(f, g) = \{(\pm 1, \pm 1)\}, f = u^2 + ay^2 - a - 1, g = au^2 + y^2 - a - 1, a \neq \pm 1$

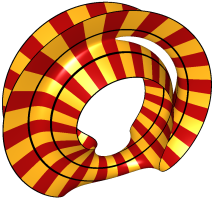


Figure : $a = -1/2$

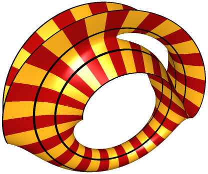


Figure : $a = 0$

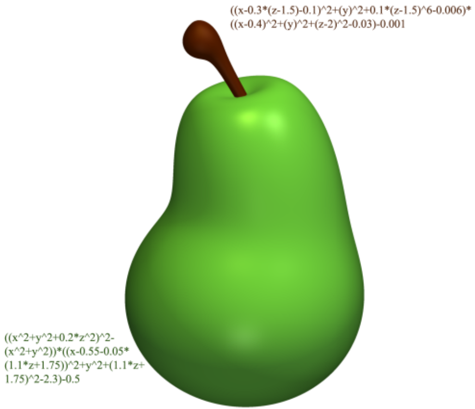
Figure : $a = 1/2$

Fruits

Competition of users during the exhibition Imaginary

Fruits

Competition of users during the exhibition Imaginary



Fruits

Competition of users during the exhibition IMAGINARY