Anexa II / la contractul de finanțare
Nr…….. 7/ 31.05.2012 ……..

SCIENTIFIC REPORT

1. **The scientific content of the event** (achieved objectives, presentations,conclusions/results obtained, contribution to further developments in the approached research field)

Commutative Algebra has experienced a striking evolution over the last decades. During that period new connections to other areas have been established and powerful techniques have been developed. A significant example of such encoding is the translation of an abstract finite simplicial complex into an ideal represented by the zeroes of square-free monomials in a set of variables corresponding to the vertices of the simplicial complex. This aspect is just one of the key interactions between Commutative Algebra and Combinatorics and it has numerous applications both within mathematics and outside.

New methods have evolved out of an influx of ideas from such diverse areas as polyhedral geometry, theoretical physics, representation theory, homological algebra, symplectic geometry, graph theory, integer programming, symbolic computation, and statistics.

The innovative goal of the school was achieved by:
- the presentation of the newest research results in the following directions: Stanley conjecture and bounds for the Stanley depth, minimal resolutions of polynomial ideals - combinatorial techniques and applications, semigroup rings and toric ideals, applications of commutative algebra and of combinatorial and computational techniques in algebraic geometry and topology.
- the presentation and discussions on some open problems in the above directions.

Of great interest is a conjecture of Stanley (1982) which relates a combinatorial inavariant of a multigraded module over the polynomial ring that is called sdepth to the depth of the module which is a homological invariant. This conjecture is still widely open. It was proved for some special cases of multigraded modules and a large number of research papers have been published on this topic.

Hilbert depth is a new invariant which was introduced recently by Bruns et al and which was helpful in studying bounds for sdepth. Recent progresses in this research topic were presented in our school in the talks of **Julio Moyano** and **Muhammad Ishaq**.

Combinatorial and homological methods in commutative algebra have become very successful in the last decades. Fundamental results in recent research in these directions were discussed in the series of lectures of **Peter Schenzel**, **Hara Charalambous** and **Apostolos Thoma**. In the same wide area may be included the lectures of **Ryota Okazaki**, **Bogdan Ichim**, **Dumitru Stamate**, **Asia Rauf**, **Alin Stefan**, **Anda Olteanu**.

The study of singularities which appear in algebraic geometry provide a rich source of nice problems and questions in commutative algebra. This was illustrated in the the talks of **Martina Kubitzke**, **Daniel Brinkmann** and **Ramakrisha Nanduri**.

There has been much activity in the last ten years in creating and implementing algorithms in commutative algebra and algebraic geometry. Many of these algorithms were originally designed for abstract algebraic geometry, but now have the potential to be used in applications.Such kind of apllications were presented in several talks by **Bogdan Ichim**, **Hara Charalambous** and **Apostolos Thoma**, **Peter Schenzel**.

Algorithms in commutative algebra and algebraic geometry go hand in hand with software packages that implement them. In the tutorials and computer demo session it was presented the Computer System Algebra **Singular** and **Normaliz** package. Several examples were discussed.

The above attractive topics, where algebraic, geometric, homological, and combinatorial tools are combined, have been presented in our school. Many questions remain open and are possible subjects for further research. Some of them were discussed in the special session on open questions.

The school has gathered together high level experts in commutative algebra, computer algebra, homological algebra, and from related fields as well (algebraic geometry, topology, etc). The school introduced Romanian students and young researchers to combinatorial commutative and computational algebra and enabeled them with the required knowledge for doing significant research in this area of mathematics. The project brought together theoreticians and practitioners interested in the development of homological and combinatorial methods in commutative algebra. On the other hand, the presentations and the computer demos together with the discussing sessions organized in addition to the program of the talks enabled all the participants, especially the students, to use these methods in their research.

Beyond the scientific profit in the benefit of the participants, one important merit of the project was that it offered to the young researchers the opportunity to create strong connections with one another, with the leading scientists and would foster long term collaborations, which would extend well beyond the present advanced school.

We now present a short abstract of the talks given in the school. For some of the presentations, the speakers made available the .pdf files with their slides. These can be found at http://gta.math.unibuc.ro/~dumi/mangalia2012/

Daniel Brinkmann, *Hilbert-Kunz functions of surfaces of type ADE*
The surfaces of type ADE are exactly the simple singularities of the form $k[x,y,z]/(f)$, where k is an algebraically closed field of characteristic at least 7 and $f \in (x,y,z)^2$. Moreover, these are exactly the rings of finite Cohen-Macaulay type (there are up to isomorphism only finitely many indecomposable maximal Cohen-Macaulay modules).
If I is an $\mm$-primary ideal in a two-dimensional local Noetherian normal domain $(R,\mm)$, the geometric approach to Hilbert-Kunz theory of Brenner and Trivedi tells us that one needs to control all Frobenius-Pullbacks of the first syzygy bundle Syz$_C(I)$ over $C:=$Proj(R) to compute the Hilbert-Kunz function of R with respect to I.
In this talk we will see how the relationship between maximal Cohen-Macaulay modules over hypersurfaces and matrix factorizations can be used to control these pullbacks in the situation of surfaces of type ADE.

Hara Charalambous and Apostolos Thoma, *Toric ideals and graphs*
Let $A=\{{\bf a}_1,\ldots,{\bf a}_m\}\subseteq \mathbb{N}^n$ be a vector configuration in $\mathbb{Q}^n$ and $\mathbb{N}A:=\{l_1{\bf a}_1+\cdots+l_m{\bf a}_m \ | \ l_i \in \mathbb{N}\}$ the corresponding affine semigroup. We grade the polynomial ring $\mathbb{K}[x_1,\ldots,x_m]$ over any field $\mathbb{K}$ by the semigroup $\mathbb{N}A$ setting $\deg_{A}(x_i)={\bf a}_i$ for $i=1,\ldots,m$. For ${\bf u}=(u_1,\ldots,u_m) \in \mathbb{N}^m$, we define the A-{\em degree} of the monomial ${\bf x}^{{\bf u}}:=x_1^{u_1} \cdots x_m^{u_m}$ to be \[\deg_{A}({\bf x}^{{\bf u}}):=u_1{\bf a}_1+\cdots+u_m{\bf a}_m \in \mathbb{N}A.\] The {\em toric ideal} I_{A} associated to A is the prime ideal generated by all the binomials ${\bf x}^{{\bf u}}- {\bf x}^{{\bf v}}$ such that $\deg_{A}({\bf x}^{{\bf u}})=\deg_{A}({\bf x}^{{\bf v}})$. Toric ideals are binomial ideals.
Let G be any graph and consider the vector configuration of the columns of the incidence matrix of G. The corresponding toric ideal I_G is of particular interest.
We are going to discuss some special sets of binomials, like circuits, Graver basis, Markov bases, the Universal Markov basis,and the Universal Gr{\"o}bner basis for toric ideals in general and for toric ideals of graphs in particular.

Mihai Epure, *A new class of pseudo-Dedekind domains and its star operations extensions*
We study the integral domains D satisfying the following condition: whenever $I>AB$ with I,A,B nonzero ideals, there exist ideals $A'>A$ and $B'>B$ such that $I=A'B'$. It turns out that D is a Pr\"ufer pseudo-Dedekind domain of dimension $1.$ Then the concept is extended in the star operation setting where the Krull dimension is no longer capped. This is joint work with Tiberiu Dumitrescu and Zaheer Ahmad.

Bogdan Ichim, *Rational polytopes in combinatorial voting theory*
We describe the use of pyramid decomposition in Normaliz, a software tool for the computation of Hilbert bases and enumerative data of rational cones and affine monoids. Pyramid decomposition in connection with efficient parallelization and streamlined evaluation of simplicial cones has enabled Normaliz to process big triangulations that arise in the computation of Hilbert series related to combinatorial voting theory.

Muhammad Ishaq, *Stanley depth and sequentially Cohen-Macaulay lexsegment ideals*
We show that if $I\subset J$ are monomial ideals of a polynomial algebra S over a field, then the Stanley depth of J/I is smaller or equal to the Stanley depth of $\sqrt{J}/\sqrt{I}$. We give also an upper bound for the Stanley depth of the intersection of two primary ideals. We show that the Stanley depth of I is less than or equal to the Stanley depth of any prime ideal associated to S/I. Also we show that Stanley's conjecture holds for I and S/I when the associated prime ideals of S/I are generated by pairwise disjoint sets of variables. We give an upper bound for the Stanley depth of the edge ideal of a complete k-partite hypergraph and as an application we give an upper bound for the Stanley depth of a monomial ideal in a polynomial ring S. We give a lower and an upper bound for the cyclic module S/I associated to the complete k-partite hypergraph. We determine the associated primes of an arbitrary lexsegment ideal $I\subset S$. As application we show that S/I is a pretty clean module, therefore, S/I is sequentially Cohen-Macaulay and satisfies Stanley's conjecture.

Martina Kubitzke, *Lefschetz properties of Artinian algebras and Hilbert series*
In this talk we introduce different kinds of Lefschetz elements of standard graded Artinian k-algebras. We explain how those elements are useful in order to derive special properties of the Hilbert series of the considered algebras, such as unimodality, being an M-sequence etc.
In the second part, we will give several applications of the introduced ideas e.g., to barycentric subdivisions of simplicial complexes, Veronese algebras and if time permits to edgewise subdivisions of simplicial complexes.

Julio Jose Moyano-Fernandez, *Hilbert depth and numerical semigroups*
Let F be a field, and consider the positively $\mathbb{Z}$-graded polynomial ring $R=F[X,Y]$. In this talk we will briefly discuss the role played by numerical semigroups in the characterization of Hilbert series of finitely generated graded R-modules of positive depth.

Ramakrishna Nanduri, *Combinatorial formulas for the Hilbert coefficients of Schubert varieties in Grassmannians*
In this talk, we give combinatorial formulas for the Hilbert coefficients, h-polynomial and the Cohen-Macaulay type of Schubert varieties in Grassmannians in terms of the posets associated with them. As a consequence, combinatorial criteria are given for a Schubert variety to be a complete intersection, Gorenstein and almost Gorenstein respectively. Finally we derive combinatorial formulas for the Hilbert coefficients of the local rings of Schubert varieties in Grassmannians at certain torus fixed points.

Ryota Okazaki, *Construction of minimal free resolutions of some monomial ideals by algebraic discrete Morse theory*
This is essentially a short survey on the recent work, due to E. Sk\"oldberg (arXiv: 1106.1913). In this talk, we will introduce a (not necessarily minimal) graded free resolution of multigraded modules over a polynomial ring which is possibly new. (the connection between our construction and Tchernev's in 2007 is not known).
As an application, we will construct a minimal graded free resolution of a monomial ideal with linear quotients. The key tool to construct is algebraic discrete Morse theory due to M. J\"ollenbeck, V. Welker, and E. Sk\"oldberg. This is a generalization of minimal graded free resolutions due to J. Herzog and Y. Takayama.

Anda Olteanu, *Powers of lexsegment ideals*
All powers of lexsegment ideals with linear resolution (equivalently, with linear quotients) have linear quotients with respect to suitable orders of the minimal monomial generators. For a special subclass of these ideals, we give a complete description of the minimal graded free resolution of their powers. We also find other classes of monomial ideals with linear quotients whose powers have linear quotients, too.

Asia Rauf, *Constructions of Cohen-Macaulay binomial edge ideals*
Let $S = K[x_1,\ldots, x_n, y_1,\ldots, y_n]$ be the polynomial ring in $2n$ variables with coefficients in a field K. Let G be a graph on vertex set $[n]$. For each edge $\{i,j\}$ of G with $i < j$, we associate a binomial $f_{ij} = x_iy_j - x_jy_i$. The ideal J_G of S generated by $f_{ij} = x_iy_j - x_jy_i$ such that $i<j$, is called \textit{the binomial edge ideal} of G.
We discuss algebraic and homological properties of binomial edge ideals associated with the graphs which are obtained by gluing of subgraphs and the formation of cones.
This is a joint work with Giancarlo Rinaldo.

Raman Sanyal, *The entropic discriminant*
The analytic centers of deformations of a real linear hyperplane arrangement constitute a family of algebraic varieties in the data of the arrangement and the deformation. Analytic centers feature in linear programming, statistics, and theoretical neuroscience and relate to constructions in classical algebraic geometry such as polar maps. The locus in the deformation space for which the variety of analytic centers is singular is given by the vanishing of a polynomial -- the entropic discriminant -- and describes the complex branch locus of the polar map associated to the hyperplane arrangement. We will survey the geometric features of the entropic discriminant and their relations to discrete invariants of the hyperplane arrangement. If time permits, we will discuss the codimension 1 case and its real algebraic implications.

Peter Schenzel, *Endomorphism rings of local cohomology modules*
Let I be an ideal of a local ring $(R,\mathfrak m)$ with $d = \dim R.$ For the local cohomology module $H^i_I(R)$ it is a well-known fact that it vanishes for $i > d$ and is an Artinian R-module for $i = d.$ In the case that the Hartshorne-Lichtenbaum Vanishing Theorem fails, that is $H^d_I(R) \not= 0,$ we explore its fine structure. In particular, we investigate its endomorphism ring and related connectedness properties. In the case R is complete we prove - as a technical tool - that $H^d_I(R) \simeq H^d_{\mathfrak m}(R/J)$ for a certain ideal $J \subset R.$ Thus, properties of $H^d_I(R)$ and its Matlis dual might be described in terms of the local cohomology supported in the maximal ideal. (joint work with Majid Eghbali.)

Let $(R, \mathfrak m)$ denote an n-dimensional Gorenstein ring. For an ideal $I \subset R$ of height c we are interested in the endomorphism ring $B = \Hom_R(H^c_I(R), H^c_I(R)).$ It turns out that B is a commutative ring. In the case of $(R,\mathfrak m)$ a regular local ring containing a field B is a Cohen-Macaulay ring. Its properties are related to the highest

Lyubeznik number $l = \dim_k \Ext_R^d(k,H^c_I(R)), d = \dim R/I.$ In particular $R \simeq B$ if and only if $l = 1.$ Moreover, we show that the natural homomorphism $\Ext_R^d(k, H^c_I(R)) \to k$ is non-zero. Further investigations concern a generalization of the Lyubeznik numbers.

Peter Schenzel, *How to visualize the blow up of points in the plane?*
Blowups are an important technique in algebraic geometry, which allow to smooth a singular algebraic variety. It is a challenge to visualize this process even in the case of blowups of points in the affine plane $X \subset {\mathcal A}^2_\mathbb{R}$. First results were obtained by M. Brodmann with the aid of the so-called toroidal blowup, a compact embedding of the blowup into affine 3-space. In fact, Brodmann provides a rational parametrization of the toroidal blowup. Unfortunately, the visualization does not work in the neighborhood of $P \in X$ because the parametrization tends to indefinite terms of the form $\frac{0}{0}$. Our approach is based on implicitization of the parametric form. By methods from commutative algebra we are able to reduce the implicitization to the computation of a single, fairly simple resultant. This provides an algebraic equation of the implicit surface of the toroidal blowup including the exceptional fiber associated with X.Surprisingly, this equation has a rather low degree. By applying additional clipping techniques to the implicit surface we are able to visualize the toroidal blowup as well as its deformations by several parameters interactively in real time. The practical aspect of the visualization is build upon the tool \textsc{RealSurf}, designed for the interactive visualization of implicit algebraic surfaces. The methods of the paper provide insights in the structure of blowups of points, even if the points are interactively moved or tend to degenerations. (joint work with Ch. Stussak.)

Dumitru Stamate, *Shellings for semigroups*
We consider the divisorial poset structure on a (numerical) semigroup and study the topology of its open intervals. We give classes of examples where these intervals are (nonpure) shellable. This has consequences in the study of the (nongraded) Koszul property of the semigroup algebra.

Alin Stefan, *The type computation of some classes of base rings*
We give the formulas for computing the type of the base ring associated to a product of transversal polymatroids.

2. Information concerning the organization of the event

The topic of the advanced school: ***Discrete invariants in commutative algebra and in algebraic geometry***

Date and place: Mangalia, Hotel Paradiso, September 2-8, 2012, September 2- arrival day, September 8- departure day.

Number of participants – 50. Number of speakers- 16: 11 from abroad (Greece, Germany, India, Italy, Japan, Pakistan) and 5 from Romania (Bucharest University, Institute of Mathematics "Simion Stoilow" of the Romanian Academy (IMAR), Ovidius University of Constanta, University of Ploiesti).

Scientific committee: Prof. Winfried Bruns (Osnabrueck, Germania), Prof Peter Schenzel (Halle, Germania) si Prof. Dorin Popescu (Bucuresti, Romania)

Organizing committee: Prof. Dorin Popescu (IMAR-Bucuresti), Prof. Viviana Ene (Univ. Ovidius-Contanta), Dr. Bogdan Ichim (IMAR-Bucuresti), Dr. Mircea Cimpoeas (IMAR-Bucuresti) si Asist. Dr. Dumitru Stamate (Univ. din Bucuresti).

Lect. dr. Denis Ibadula (Univ. Ovidius-Constanta) and Mihai Epure (IMAR-Bucuresti) joined the organizing team in Mangalia.

The Scientific Committee led the scientific debates and the open questions session. In addition, the Organizing Committee oversaw local matters such as publicity, multimedia and the day-to-day handling of the budget. The following material was presented at the school: posters, list of participants, program of the talks, abstracts.

Advertizing of the school:

The web page of the project has been prepared before the beginning of the event. It contains special sections for information concerning the organization and program of the school. The web page has been hosted by Faculty of Mathematics and Computer Science, Ovidius University at the address: http://math.univ-ovidius.ro/sna/edition.aspx?cat=GeneralInfo&itemID=6

The school was also advertised through correspondence with Romanian colleagues and on the following websites: Faculty of Mathematics and Computer Science, Bucharest University, Institute of Mathematics, Bucharest, and www.commalg.org (website for the commutative algebra community).
The poster of the school was created by Rosu Ciprian George. It was posted in various universities from Romania.

Selection of the speakers:
The scientific programme an the selection of the speakers was mainly the duty of the scientific committee. Compared to the initial grant proposal, this committee was extended to include prof. W. Bruns from Univ. Osnabrueck, Germany. All the confirmed speakers from the initial grant application have successfully attended the School.

Selection of the participants:
Besides the 11 keyspeakers from abroad, there were 39 participants from Romania: researchers, PhD students and students. The advertising of the School, as described above, proved to be very efficient. We would like to emphasize that more than half of the Romanian participants were university students, M.Sc and PhD students or recent PhDs.

Financial aspects:
The promptitude of the sponsoring agency CNCS-UEFISCDI in evaluating our grant proposal and in the phase of signing the grant allowed more time for the selection of the keyspeakers and for finding a proper location where the School could take place. Guests from abroad were able to find plane tickets with more convenient prices. Also, several of them had financial support for travel from their home institutions.
All these resulted in less money being spent on travel and housing than foreseen at the time of the application. Thus, we were able to support more Romanian participants (including students) than estimated.

Reports:
The scientific report of this school will be available online at
http://gta.math.unibuc.ro/~dumi/mangalia2012

3. **The final program of the event**

Registration: Sunday, 2nd September, 16:00-18:00, Hotel Paradiso, Mangalia

Monday		
9:00-10:00	P. Schenzel	Endomorphism rings of local cohomology modules (I)
10:00-10:30	Coffee break	
10:30-11:20	R. Sanyal	The entropic discriminant
11:40-12:30	J. Moyano	Hilbert depth and numerical semigroups
15:30-15:55	A. Stefan	The type computation of some classes of base rings
16:05-16:30	A. Olteanu	Powers of lexsegment ideals
16:40-17:30	Tutorial/Computer Demo (Ene)	

Tuesday		
9:00-10:00	Charalambous/Thoma	Toric ideals and graphs I
10:00-10:30	Coffee break	
10:30-11:30	P. Schenzel	Endomorphism rings of local cohomology modules (II)
11:40-12:30	R. Okazaki	Construction of minimal free resolutions of some monomial ideals by algebraic discrete Morse theory
15:30-15:55	D. Stamate	Shellings for semigroups
16:05-16:30	M. Epure	A new class of pseudo-Dedekind domains and its star operations extensions
16:40-17:30	Tutorial (Stamate)	

Wednesday		
9:00-10:00	Charalambous/Thoma	Toric ideals and graphs II
10:00-10:30	Coffee break	
10:30-11:20	M. Kubitzke	Lefschetz properties of Artinian algebras and Hilbert series
11:30-12:20	D. Brinkmann	Hilbert-Kunz functions of surfaces of type ADE
12:30-13.20	Ramakrishna Nanduri	Combinatorial formulas for the Hilbert coefficients of Schubert varieties in Grassmannians

Thursday		
9:00-10:00	P. Schenzel	How to visualize the blow up of points in the plane?
10:00-10:30	Coffee break	
10:30-11:30	Charalambous/Thoma	Toric ideals and graphs III
11:40-12:30	A. Rauf	Constructions of Cohen-Macaulay binomial edge ideals
15:30-16:20	Tutorial (Charalambous/Thoma)	Toric ideals and graphs
16:30-17:20	Open problems/ Discussions	

Friday		
9:00-10:00	Charalambous/Thoma	Toric ideals and graphs IV
10:00-10:30	Coffee break	
10:30-11:30	B. Ichim	Rational polytopes in combinatorial voting theory
11:40-12:30	M. Ishaq	Stanley depth and sequentially Cohen-Macaulay lexsegment ideals

4. List of participants

1 Ioana Amarande (Bucharest University, Romania)
2 Florin Ambro (Institute of Mathematics, Bucharest, Romania)
3 Cristina Anghel (Bucharest University, Romania)
4 Eduard Asadurian (Pitesti University, Romania)
5 Serban Barcanescu (Institute of Mathematics, Bucharest, Romania)
6 Serban Basarab (Institute of Mathematics, Bucharest, Romania)
7 Wladimir Boskoff (Ovidius University, Romania)
8 Daniel Brinkmann (Osnabrueck University, Germany)
9 Hara Charalambous (Thessaloniki University, Greece)
10 Mihai Cipu (Institute of Mathematics, Bucharest, Romania)
11 Andrei Cocalea (Bucharest University, Romania)
12 Adrian Constantinescu (Institute of Mathematics, Bucharest, Romania)
13 Ahmet Dokoyucu (Ovidius University, Romania)
14 Viviana Ene (Ovidius University, Romania)
15 Mihai Epure (Institute of Mathematics, Bucharest, Romania)
16 Petru Hlihor (Bucharest University, Romania)
17 Denis Ibadula (Ovidius University, Romania)
18 Bogdan Ichim (Institute of Mathematics, Bucharest, Romania)
19 Irina Ilioaea (Bucharest University, Romania)
20 Cristodor Ionescu (Institute of Mathematics, Bucharest, Romania)
21 Gabriel Iorgulescu (Ovidius University, Romania)
22 Muhammad Ishaq (National University of Science and Technology, Islamabad, Pakistan)
23 Daniel Ivan (Institute of Mathematics, Bucharest, Romania)
24 Martina Kubitzke (Goethe Universitaet, Frankfurt am Main, Germany)
25 Vlad Margarint (Bucharest University, Romania)
26 Julio Jose Moyano-Fernandez (Osnabrueck University, Germany)

27 Ramakrishna Nanduri (The Institute of Mathematical Sciences, Chennai, India)
28 Ryota Okazaki (Osaka University, Japan)
29 Oana Olteanu (Ovidius University, Romania)
30 Anda Olteanu (Ovidius University, Romania)
31 Alexandra Otiman (Bucharest University, Romania)
32 Ovidiu Pasarescu (Institute of Mathematics, Bucharest, Romania)
33 Alexandra Peste (Bucharest University, Romania)
34 Dorin Popescu (Institute of Mathematics, Bucharest, Romania)
35 Steluta Pricopie (Bucharest University, Romania)
36 Asia Rauf (ICTP, Trieste, Italy)
37 Miruna Rosca (Bucharest University, Romania)
38 Raman Sanyal (Freie Universitaet, Berlin, Germany)
39 Peter Schenzel (Martin-Luther-Universitaet Halle-Wittenberg, Germany)
40 Andrei Sipos (Bucharest University, Romania)
41 Dumitru Stamate (Bucharest University, Romania)
42 Miron Stanciu (Bucharest University, Romania)
43 Alin Stefan (Ploiesti University, Romania)
44 Mirela Stefanescu (Ovidius University, Romania)
45 George Teseleanu (Bucharest University, Romania)
46 Apostolos Thoma (Ioannina University, Greece)
47 Radu Titiu (Bucharest University, Romania)
48 Laurentiu Silviu Vasile (Bucharest University, Romania)
49 Cristian Voica (Bucharest University, Romania)
50 Andrei Zarojanu (Bucharest University, Romania)

Project director,

Prof. Dr. Dorin POPESCU

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